Lecture 5: Observation Impact & Observation Sensitivity
Outline

- Observation impacts
- Adjoint 4D-Var: $(4D-\text{Var})^T$
- Observation sensitivity
- Error covariance estimates from $(4D-\text{Var})^T$
Observation Impacts

(Useful references: Langland & Baker, 2004; Gelaro and Zhu, 2009; Tremolet, 2008)
Given the plethora of different observation platforms, what impact does each have on the 4D-Var analysis?
Observation Impacts

Consider a scalar function of the ocean state vector:

\[ I = I(x) \]

Prior: \[ I_b = I(x_b) \]

Posterior: \[ I_a = I(x_a) \]

Increment: \[ \Delta I = I_a - I_b \]

\[ x_a(t) = x_b(t) + \delta x(t) \]

\[ I_a = I(x_b + \delta x) \approx I_b + \delta x^T \ast (\partial I/\partial x) \]

\[ \Delta I \approx \delta x^T \ast (\partial I/\partial x) \]

but \[ \delta x(t) = M_b(t, t_0) \tilde{K}d \]

\[ \Delta I \approx d^T \tilde{K}^T M_b^T \ast (\partial I/\partial x) \]

\((M_b^T \ast (\partial I/\partial x)\) denotes a time convolution)
Consider a scalar function of the ocean state vector:

\[ I = I(x) \]

Prior: \[ I_b = I(x_b) \]

Posterior: \[ I_a = I(x_a) \]

Increment: \[ \Delta I = I_a - I_b \]

\[ \Delta I \approx d^T \tilde{K}^T M^T * (\partial I / \partial x) \]

- Innovations
- Adjoint of gain matrix
- Adjoint model
Observation Impacts

Recall the dual form of the gain matrix:

\[ K \approx \tilde{K}_k = DG^T R^{-1/2} V_k T_k^{-1} V_k^T R^{-1/2} \]

So:

\[ \tilde{K}_k^T = R^{-1/2} V_k T_k^{-1} V_k^T R^{-1/2} GD \]

Therefore:

\[ \Delta I \approx d^T R^{-1/2} V_k T_k^{-1} V_k^T R^{-1/2} GDM^T * (\partial I / \partial x) \]
Sequential 4D-Var with 10km CCS ROMS

Observations

4D-Var Analysis

Posterior

Forecast

prior

Observations

4D-Var Analysis

Posterior

Forecast

prior

Observations

4D-Var Analysis

Posterior

Forecast

prior

7 days
Sequential 4D-Var CCS ROMS

10km, I4D-Var, 1X10

10km, I4D-Var, 1X10

log$_{10}$(J)

07/27/02 07/27/03 07/26/04

J initial

J final
Example: 37N Transport

10km, CCS ROMS
Example: 37N Transport

No assim

Primal Strong

JAS time mean alongshore Flow (10km, 42 lev)

CC = California Current
CUC = California Under Current
37N Transport Observation Impacts

The time average 37N transport can be written as:

$$I_{37N} = \frac{1}{N} \sum_{i=1}^{N} h^T x_i$$

where: \( x_i \equiv x(i\Delta t) = x(t) \)

therefore:

$$\Delta I_{37N} = \frac{1}{N} \sum_{i=1}^{N} h^T \left( (x_a)_i - (x_b)_i \right)$$

$$\approx \frac{1}{N} \sum_{i=1}^{N} h^T (M_b)_i \tilde{K}d = d^T \tilde{K} \sum_{i=1}^{N} \frac{1}{N} (M_b)_i h$$

where: \((M_b)_i \equiv M(t_0 + i\Delta t, t_0) = M(t, t_0)\)
37N Transport Observation Impacts

37N time averaged transport increment:

\[
\Delta I_{37N} \approx \frac{1}{N} d^T \tilde{K}^T \sum_{i=1}^{N} (M_b)_i^T h
\]

\[
\tilde{K}_k^T = R^{-1/2} V_k^T \tilde{T}_k^{-1} V_k^T R^{-1/2}
\]

- ADROMS forced by \( h \)
- TLROMS sampled at observation points
- Dual space Lanczos vectors
37N Transport

\[ J \text{ (Sv)} \]

*Prior transport*

\[ J \text{ (Sv)} \]

*Transport increment*

- **NL increment**
- **TL Increment**
**Control Vector Impacts**

37N time averaged transport increment:

\[ \Delta I_{37N} \approx \frac{1}{N} d^T \tilde{K}^T \sum_{i=1}^{N} (M_b)_i^T h \]

\[ = d^T g = d^T \left( g_x + g_f + g_b \right) \]

where:

\[ g \approx \frac{1}{N} \tilde{K} \sum_{i=1}^{N} (M_b)_i^T h \]

- \( g_x \) - contribution from initial condition increments
- \( g_f \) - contribution from surface forcing increments
- \( g_b \) - contribution from open boundary increments
37N Transport Control Vector Impacts

\[ \Delta I \text{ (Sv)} \]

- Initial condition
- Forcing
- Boundary conditions

rms
Observation Impacts

37N time averaged transport increment:

\[
\Delta I_{37N} \approx \frac{1}{N} d^T \tilde{K}^T \sum_{i=1}^{N} (M_b)_i^T h
\]

\[
= d^T g = \sum_{i=1}^{N_{obs}} d_i g_i
\]

\[
= \sum_{i=1}^{N_{obs}} \left( y_i - H_i \left( x_b(t) \right) \right) g_i
\]

Contribution of each observation to \( \Delta I \)
37N Transport Observation Impacts
37N Transport Observation Impacts

SSH

SST

Argo S

CTD S
Two Spaces: Obs Impact

$K$ maps from observation (dual) space to model (primal) space

$K^T$ maps from model (primal) space to observation (dual) space

Identifies the part of model space that controls 37N transport and that is activated by the observations
Observation Impacts: ROMS Implementation

• Primal (I4D-Var) and dual (4D-PSAS & R4D-Var) forms available:
  - define IS4DVAR_SENSITIVITY
    Drivers/obs_sen_is4dvar.h

  - define W4DPSAS_SENSITIVITY
    define OBS_IMPACT
    define OBS_IMPACT_SPLIT
    Drivers/obs_sen_w4dpsas.h

  - define W4DVAR_SENSITIVITY
    define OBS_IMPACT
    define OBS_IMPACT_SPLIT
    Drivers/obs_sen_w4dvar.h
Adjoint 4D-Var & Observation Sensitivity
How will the circulation analysis change if some of the observations or the observation array change?
Adjoint 4D-Var & Observation Sensitivity

The analysis increments are a nonlinear function of the innovation vector \( d \):

\[
\mathbf{z}_a = \mathbf{z}_b + K(d)
\]

where:

\[
d = \mathbf{y} - H\left(\mathbf{z}_b(t)\right)
\]

Consider variations in the observation vector \( \delta y \):

\[
\delta d = \delta y;
\quad \mathbf{z}_a + \delta \mathbf{z}_a = \mathbf{z}_b + K(d + \delta d)
\]

\[
\delta \mathbf{z}_a \approx \frac{\partial K}{\partial \mathbf{y}} \delta \mathbf{y}
\]

Tangent linearization of 4D-Var
Adjoint 4D-Var & Observation Sensitivity

Consider a scalar function of the posterior control vector $z_a$:

$$
I_a = I(z_a) = I(z_b + K(d))
$$

A change $\delta y$ in the observations yields a change in $\Delta I_a$:

$$
I_a + \Delta I_a = I(z_b + K(d + \delta y))
\approx I(z_b + K(d) + (\partial K/\partial y) \delta y)
\approx I(z_a) + ((\partial K/\partial y) \delta y)^T (\partial I/\partial z)
$$

Therefore:

$$
\Delta I_a \approx \delta y^T (\partial K/\partial y)^T (\partial I/\partial z)
$$
Adjoint 4D-Var & Observation Sensitivity

\[ \Delta I_a \approx \delta y^T \left( \frac{\partial K}{\partial y} \right)^T \left( \frac{\partial I}{\partial z} \right) \]

Adjoint of 4D-Var
Observation System Experiments (OSEs)

Suppose that during a particular assimilation cycle the satellite altimeter goes offline.

How would this have impacted the analysis?

We could run 4D-Var again with SSH obs removed.

Or let $\delta y_i = -d_i$ for all SSH obs.

The change in the analysis is:

$$\delta z_a \approx \left( \partial K / \partial y \right) \delta y$$

The change in $\Delta I_a$ is:

$$\Delta I_a \approx \delta y^T \left( \partial K / \partial y \right)^T \left( \partial I / \partial z \right)$$
Observation System Experiments (OSEs)

The cost of $(4D\text{-}Var)^T = \text{cost of } 4D\text{-}Var$

But ONLY one run of $(4D\text{-}Var)^T$ is needed for ALL OSEs.
Example: 37N transport

\( \Delta I = d^T \tilde{K}^T \sum_{i=1}^{N} \frac{1}{N} (M_b)_i^T h_i \)

\( \Delta I = d^T \left( \frac{\partial K}{\partial y} \right)^T \sum_{i=1}^{N} \frac{1}{N} (M_b)_i^T h_i \)
Sequential 4D-Var with 30km CCS ROMS

Observations → 4D-Var Analysis → Posterior

prior

4D-Var Analysis → Posterior

prior

4D-Var Analysis → Posterior

prior

7 days

Forecast

Forecast

Forecast
Sequential 4D-Var CCS ROMS

30km, R4D-Var, 1X50

$log_{10}(J)$

Date

J initial

J final
Observing System Experiments (OSEs)
(30km, CCS ROMS)

Change in 37N transport when Argo withheld

Change in 37N transport when SSH withheld
Two Spaces: Obs Sensitivity

\( \partial K / \partial y \) maps from observation (dual) space to model (primal) space

\((\partial K / \partial y)^T\) maps from model (primal) space to observation (dual) space

*Identifies the part of model space that controls 37N transport and that is activated by the observations during 4D-Var*
Observation Sensitivity: ROMS Implementation

- Dual (4D-PSAS & R4D-Var) forms only available:

  - define W4DPSAS_SENSITIVITY
    (define RECOMPUTE_4DVAR)
    Drivers/obs_sen_w4dpsas.h

  - define W4DVAR_SENSITIVITY
    (define RECOMPUTE_4DVAR)
    Drivers/obs_sen_w4dvar.h
Error Covariance Estimates from \((4D-\text{Var})^T\)
Analysis Error Revisited

State-vector $\mathbf{x}$

$\mathbf{y} + \delta \mathbf{y}_l$

$\mathbf{z}_b + \delta \mathbf{z}_l$

Ensemble covariance = Analysis covariance

Ensemble of $M$ 4D-Var analyses


\[
\mathbf{E}^a = \frac{1}{M} \sum_{l=1}^{M} \left( \mathbf{x}_l^a - \mathbf{x}^a \right) \left( \mathbf{x}_l^a - \mathbf{x}^a \right)^T
\]
Analysis Error Revisited

An ensemble of 4D-Var analyses is very expensive!

But one run of (4D-Var)$^T$ yields $\left(\frac{\partial K}{\partial d}\right)^T$ and:

$$\delta z^a_i \approx \delta z_i + \left(\frac{\partial K}{\partial d}\right) \delta d_i; \quad \delta d_i \approx \delta y_i + G \delta z_i$$

and: $\delta x^a_i(t) \approx M(t,t_0) \delta z^a_i$

Therefore:

$$E^a_x(t) = \left\langle \delta x^a(t) \left(\delta x^a(t)\right)^T \right\rangle$$

$$= M \left\{ \left( I - \left(\frac{\partial K}{\partial d}\right) G \right) D \left( I - \left(\frac{\partial K}{\partial d}\right) G \right)^T + \left(\frac{\partial K}{\partial d}\right) R \left(\frac{\partial K}{\partial d}\right)^T \right\} M^T$$

Here, $M$ is essentially infinite!
Analysis Error Revisited

For linear functions:  \[ I(x) = \sum_{k=1}^{N} h_k^T x_k \]

*Posterior* analysis error variance:

\[
\begin{align*}
\left( \sigma_I^a \right)^2 &= \left( \sum_{k=1}^{N} h_k^T \mathcal{M}_k \right) E_x^a(t_0) \left( \sum_{j=1}^{N} \mathcal{M}_j^T h_j \right) \\
&= g^T E_x^a(t_0) g
\end{align*}
\]

where: \[ g = \sum_{j=1}^{N} \mathcal{M}_j^T h_j \] (ADROMS forced by h)
Sequential 4D-Var with 30km CCS ROMS

Observations → 4D-Var Analysis → Posterior → Forecast

Observations → 4D-Var Analysis → Posterior → Forecast

Observations → 4D-Var Analysis → Posterior → Forecast

prior

7 days
Example: 37N Transport

30km, ROMS CCS, 7day average transport errors

- Prior error
- Posterior error using $(4D-$Var$)^T$
- Posterior error using $\tilde{E}^a$
Predictability due to assimilating observations during \( [t_0, t_0+7] \):

\[
\left( \sigma_{14}^f \right)^2 - \left( \sigma_7^f \right)^2
\]
Example: 37N Transport Predictability

\[
\begin{align*}
\left( \sigma^f_{14} \right) &= \text{spread of 14 day forecast ensemble of transport} \\
\left( \sigma^f_7 \right) &= \text{spread of 7 day forecast ensemble of transport}
\end{align*}
\]

\[
\left( \sigma^f_{14} \right)^2 - \left( \sigma^f_7 \right)^2 = 2g^T GD \mathcal{M}_b^T \sum_k (\mathcal{M}_{14})_k^T h_k - g^T \left( GDG^T + R \right) g
\]

where:

\[
g = \left( \frac{\partial K}{\partial d} \right)^T \mathcal{M}_b^T \sum_k (\mathcal{M}_{14})_k^T h_k
\]

Seemingly complicated expressions, but really just TL and AD operators strung together in the right order!
Example: 37N Transport Predictability

\[
\left( \sigma_{14}^f \right)^2 - \left( \sigma_7^f \right)^2 = 2g^TGD\mathcal{M}_b^T \sum_k (\mathcal{M}_{14})_k^T h_k - g^T \left( GDG^T + R \right) g
\]

change in predictability due to the covariance between errors in the *priors* \( z_b \) and errors in the time evolving *prior* circulation \( x_b(t) \) evaluated at the observation points.

change in predictability associated with the stabilized representer matrix - a combination of the covariance between errors in the time evolving *prior* circulation at the observation points, and the covariance between the observation errors (including errors of representativeness).
Example: 37N Transport Predictability

\[ r = 100 \frac{\left( \sigma^f_{14} \right)^2 - \left( \sigma^f_7 \right)^2}{\left( \sigma^f_{14} \right)^2} \]

\( r > 0 \) implies 4D-Var increases predictability
Issues, Things to do, & Coming Soon

• Observation sensitivity only available for dual 4D-Var.
• Observation impact and observation sensitivity calculations are currently restricted to a single outer-loop – multiple outer-loops coming soon.
• Increase the modularity of ROMS drivers so that arbitrary sequences of operators (linear and non-linear) can be formed.
Summary

• Observation impact is based on $\tilde{K}^T$ and yields the actual contribution of each obs to the circulation increments.
• Observation sensitivity is based on $(4D$-Var$)^T$ and yields the change in circulation due to changes in obs (or array) - useful for efficient generation of OSEs.
• Both obs impact and obs sensitivity were applied in examples during analysis cycle, but can be applied during forecast cycle also (Moore et al, 2010c).
• $(4D$-Var$)^T$ yields more reliable estimates of $E^a$ and $E^f$ and predictability.
References


References

