Error evolution in the dynamics of an ocean general circulation model

Achim Wirth*, Michael Ghil

Department of Atmospheric Sciences and Institute of Geophysics and Planetary Physics, University of California, Los Angeles, CA 90095-1567, USA

Received 2 August 1998; received in revised form 24 February 1999; accepted 30 June 1999

Abstract

The problem of error propagation is considered for spatially uncorrelated errors of the barotropic stream function in an oceanic general circulation model (OGCM). Such errors typically occur when altimetric data from satellites are assimilated into ocean models. It is shown that the error decays at first due to the dissipation of the smallest scales in the error field. The error then grows exponentially before it saturates at the value corresponding to the difference between independent realizations. A simple analytic formula for the error behavior is derived; it matches the numerical results documented for the present primitive-equation ocean model, and other models in the literature. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Error evolution; Ocean general circulation model; Primitive-equation ocean model

1. Introduction

The growth of initial errors limits our ability to simulate and predict atmospheric and oceanic flows; it is thus a key issue in their better understanding and forecasting. Numerical models of the ocean may exhibit a dependence on initial states that differs from the one found in the oceanic flows’ true dynamics. We consider this possibility in the present paper.

* Corresponding author. Current address: Institut fuer Meereskunde, Duesternbrooker Weg 20, 24105 Kiel, Germany.
E-mail address: awirth@ifm.uni-kiel.de (A. Wirth).

0377-0265/00/$ - see front matter © 2000 Elsevier Science B.V. All rights reserved.
PII: S0377-0265(00)00053-1
The sea-surface height data provided by satellite altimetry are one of the most abundant and important sources of information about the ocean, especially through their assimilation into numerical models. The height errors from altimetric data are commonly assumed to be white in space, that is, completely uncorrelated from point to point, after systematic errors have been filtered out (e., Fu et al., 1994). It is thus of great interest to study the response of an ocean model to initial errors that are white in space.

The dynamics of small differences between initial states in the ocean itself and in a numerical model thereof may indeed be different, as stated above, especially when the spatial correlation of the errors is of the order of the model resolution; this is the case we shall address here. The decorrelation length scale of an error field that is spatially white in theory equals the model’s grid size in practice. This length scale is in the model’s dissipation range, where linear dynamics dominates.

The grid size of most oceanic general circulation models (OGCMs) is still quite close to, and barely smaller than, the so-called “mesoscale.” In fact, this scale equals roughly the first baroclinic Rossby radius of deformation, \( L_R \), and corresponds to the so-called “synoptic” scale in the atmosphere; it characterizes the most energetic eddy motions in the observed ocean and thus behavior at this scale is actually quite nonlinear. When parameterizing the subgrid scales by an eddy-viscosity representation — as is done in most OGCMs and in the numerical model chosen here — it is thus very unlikely that the dynamics at scales of the order of the grid size are correctly represented, in particular with the respect to the issue of error evolution.

The key point in our investigation is that the error evolution depends on the errors’ spatial correlation structure, as pointed out already by Lorenz (1969, 1982) and that, furthermore, randomly inserted small-scale errors first decay due to viscous damping and geostrophic adjustment. Initial error decay was noted in the meteorological literature in connection with numerical experiments in preparation for what became eventually known as the Global Weather Experiment (e.g., Williamson and Kasahara, 1971) and, more recently, in the oceanographic literature by Brasseur et al. (1996), among others. To the best of our knowledge, this initial decay has not yet been discussed in detail or given a quantitative description.

Much of the early error behavior can be captured by a linear model, in which the exponential rate of change — growth or decay — of the error depends on its spatial scale. At the initial time the error field possesses fluctuations of comparable magnitude on all scales, as the error field is taken to be white in space. The high-wave number part is subject to strong viscous damping and is thus attenuated in a very short time, while a subset of the low wave numbers achieves exponential growth due to barotropic or baroclinic instabilities. The net integrated effect is an initial decay and a subsequent growth of the error.

In the numerical experiments described herein, the errors are introduced in the barotropic part of the stream function. During realistic data assimilation experiments errors are also introduced into the flow fields’ baroclinic part. The distribution of the errors between barotropic and baroclinic dynamics depends, of course, on the model dynamics and the data assimilation method used. This distribution, and the effects of error growth due to internal dynamics vs. errors in the wind-stress forcing, are the topic of a separate paper (Ma et al., 2000 in preparation) and we shall not dwell on it here.
This paper is organized as follows. In Section 2, the error-evolution problem is formulated. Statistically significant results of numerical experiments are presented in Section 3. In Section 4, a simple analytic formula for the error evolution is derived, accounting for the initial decay and subsequent growth of errors observed in Section 3. In Section 5, we discuss the relevance of the present results to previous findings on error evolution and to applications like data assimilation. Concluding remarks follow in Section 6.

2. Theoretical formulation of the problem

We study error evolution in a multi-level, primitive-equation ocean model (see Section 3 for a brief description). At time $t = 0$ in the evolution of this model, we perturb its barotropic stream function as follows:

$$\tilde{\psi}(\lambda, \phi, 0) = \psi(\lambda, \phi, 0) + \epsilon(\lambda, \phi, 0),$$

where $\lambda$ is longitude, $\phi$ is latitude, $\psi(\lambda, \phi, 0) \equiv \psi(t = 0)$ is the true initial state, and $\tilde{\psi}$ is the perturbed field.

The initial stream-function error $\epsilon(\lambda, \phi, 0)$ is white in space, that is, uncorrelated between any two distinct points in space:

$$\langle \epsilon(\lambda, \phi, 0) \epsilon(\lambda', \phi', 0) \rangle_{\epsilon} = \delta(\lambda - \lambda') \delta(\phi - \phi') \eta^2 \|\psi(t = 0)\|^2;$$

here $\langle . \rangle_{\epsilon}$ denotes ensemble averaging over different realizations of $\epsilon(\lambda, \phi, 0)$, $\delta$ is the Kronecker symbol, $\eta \ll 1$ is a numerical constant measuring the standard deviation of the perturbation relative to $\|\psi(t = 0)\| = \sqrt{\langle \psi(t = 0)^2 \rangle}$, the spatial root-mean-square (rms) average of the true initial state over the whole domain. For $t > 0$, the stream-function error will also be a function of $\psi(t = 0)$, so that we have to take ensemble averages over different realizations of the true initial barotropic stream function as well. Ensemble averages over different realizations of both $\epsilon$ and $\psi$ are denoted by $\langle . \rangle_{\epsilon}$.

We are interested in the time evolution of the square root $V(t)$ of the stream function’s relative error variance:

$$V(t) \equiv \sqrt{\langle \|\epsilon(t)\|^2 / \|\psi(t)\|^2 \rangle_{\epsilon, \phi}} .$$

The square root of the stream-function error variance is commonly considered in ocean predictability experiments (Holland and Malanotte-Rizzoli, 1989; Brasseur et al., 1996). This is distinct from the total kinetic error energy $\langle (\nabla \epsilon(\lambda, \phi, t) \cdot \nabla \epsilon(\lambda, \phi, t))_{\epsilon, \phi} \rangle$, where $\nabla$ is the gradient operator. The latter is the usual measure of error in atmospheric predictability experiments, but does not exist in the limit of error fields that are truly white in space, that is, uncorrelated at vanishing separation between points. We are interested here in results that will hold up as the grid size in OGCMs continues to decrease, and wish to avoid therefore singularities in the limit of vanishing grid size.
3. Numerical results

For our numerical experiments we used a version of the OGCM introduced by Bryan (1969), which is currently available from NOAA’s Geophysical Fluid Dynamics Laboratory in its MOM2.2 version. The barotropic stream function is a prognostic variable in the particular version of the Fortran-90 code for MOM2.2 that we chose. The basin geometry is $20^\circ \times 10^\circ$ in longitude and latitude, respectively, and the maximal depth is 1500 m. The resolution is $0.25^\circ \times 0.25^\circ$ in the horizontal and 12 levels in the vertical. The rather small model domain was chosen to obtain statistically significant results by running it at the prescribed resolution, in parallel on a number of workstations.

The topographic structure in the western part of our basin resembles to a certain extent the topography of the North Atlantic near Cape Hatteras (see Fig. 1b). The vertical stratification induced by the initial temperature profile leads to a first baroclinic radius of deformation of $L_R \approx 40$ km.

The experiments are performed for two different values of horizontal viscosity, $\nu = 2.5 \cdot 10^6 \text{ cm}^2 \text{s}^{-1}$ and $3.75 \cdot 10^6 \text{ cm}^2 \text{s}^{-1}$. The vertical viscosity is $5 \text{ cm}^2 \text{s}^{-1}$ and the Prandtl number is 10 in both the vertical and the horizontal direction. The bottom drag coefficient is $c = 2.5 \cdot 10^{-3}$ (see Gill, 1982, p. 29).

The model ocean is driven by a zonal wind forcing that is constant in time and whose meridional shear (see Fig. 1a) leads to a double-gyre circulation (see Fig. 2). The model dynamics is in the chaotic regime of this circulation (Jiang et al., 1995) for all the initial states used.

A spin-up time of 1000 days suffices for the circulation to reach a statistically steady regime in such a small basin, as shown in Fig. 2. After this spin up, a random error $\epsilon(\lambda, \phi, 0)$ is added to the model’s barotropic stream function that is uncorrelated from one grid point to another. The perturbed and unperturbed (“control”) runs are then continued for up to 100 days. At discrete times the stream-function difference $V(t)$ [see Eq. (3)] between the two runs is measured, for all grid points contained in the square shown in Fig. 2a. The analysis of the error behavior is restricted to this square of $32^2$ grid points, which are far from boundaries and topographic changes.

All the experiments were performed several times using different initial states — to average over $\psi(t = 0)$ — and realizations of the initial error — to average over...
Fig. 2. Typical initial state for our perturbation experiments (after spin up, $\nu = 2.5 \times 10^6 \text{ cm}^2 \text{ s}^{-1}$): (a) Barotropic stream function; positive contours are solid, negative ones are dashed, and the units are cm$^2$ s$^{-1}$. (b) Velocity field at 40-m depth (maximal velocity $\approx 1.0 \text{ m s}^{-1}$). The large square in panel (a) marks the region within which the error field is analyzed (see text).

Fig. 3. Error evolution for 300 days. The thin solid line shows the stream function’s relative standard deviation $V(t)$, averaged over four different realizations of the initial perturbation $\epsilon(t = 0)$ for each of four unperturbed stream function fields $\psi(t = 0)$, while the two dotted lines above and below it represent the standard deviations from this mean value of $V(t)$; the heavy dashed–dotted line is the best fit of Eq. (9) [based on Eqs. (6) + (8)] to the mean; the dashed line gives the mean, over the four realizations of $\psi$, of the relative (normalized by the rms value of the stream function in the analyzing square) difference between two stream-function fields in the same experiment separated by the time interval $t$ on the abscissa.
Fig. 4. Error evolution for 25 days and different values of the viscosity, $\nu = 2.5 \cdot 10^6$ cm$^2$ s$^{-1}$ (heavy dotted line) and $3.75 \cdot 10^6$ cm$^2$ s$^{-1}$ (heavy solid line) and the best fit (corresponding thin lines) to Eq. (9).

Fig. 5. Error evolution over 50 days for two different amplitudes of the initial error, $\eta = 10^{-2}$ (solid line) and $10^{-1}$ (dotted line). Both curves are normalized by the error at day 1.

\[ e(t = 0) \]. We also used two different values for the amplitude $\eta$ of the perturbation, as well as two different values of the eddy viscosity $\nu$.

In Fig. 3, we show the mean error (light solid) and its standard deviation averaged over 16 experiments with an initial relative error variance of $\eta = 10^{-2}$ and an eddy viscosity $\nu = 3.75 \cdot 10^6$ cm$^2$ s$^{-1}$; each experiment was run for 300 days after introducing the initial error. The results clearly show an error decay for the first 6-to-12 days, followed by a rapid error growth which slows down and then reaches saturation after about 300 days. The same figure also shows the difference (light dashed) between an unperturbed stream-function field at time $t$ and its initial state (averaged over four experiments).

In Fig. 4, we compare the error behavior over the first 25 days after perturbation for the two different values of the eddy viscosity, $\nu = 2.5$ and $\nu = 3.75$, in units of $10^6$ cm$^2$. 
s$^{-1}$. One clearly sees that — for the larger value of the eddy viscosity — the initial error decay is faster, while the subsequent growth is fairly independent for the two values of $\nu$ chosen.

We performed furthermore 16 error-evolution experiments for the intermediate time interval of 50 days; in these, two different values of $\eta$, equal to $10^{-2}$ and $10^{-3}$, were used, while $\nu = 3.75 \cdot 10^6$ cm$^2$ s$^{-1}$ (see Fig. 5). By changing the relative amplitude $\eta$ of the initial error field [see Eq. (2)], we can estimate the influence of nonlinear effects, if any, on the early error dynamics: if the initial error evolution is linear, the errors for a short time will be proportional to $\eta$; otherwise they will not. In other words, we can determine by varying $\eta$ how well the error decay is represented by the tangent linear operator acting on the barotropic stream function $\psi$. To consider this issue, we normalized the square root of the error variance by $\eta$ and plotted the average error evolution for the corresponding experiments in Fig. 5.

It is important to note that the actual initial error’s variance in all the figures differs from the corresponding value of $\eta^2$. This difference arises because $\eta$ is normalized by the rms stream function over the whole basin [see Eq. (2)], while in the figures the normalization is carried out over the analyzing square only (see Fig. 2), and the rms values of $\psi$ over the whole basin and the analyzing square are slightly different. This difference is also a function of the viscosity (see Fig. 4), because by changing the viscosity $\nu$ not only the amplitude of the streamfunction changes but also its overall shape.

### 4. An analytic model for the error dynamics

The good agreement in the error evolution between different error-field realizations and experiments immediately suggests that the quantitative behavior of the error dynamics can be approximated by a model involving only a few parameters. No significant difference between the error decay for the two different values of $\eta$ can be seen in Fig. 5, and the two mean values are very well described by the same model. This shows that the decay of spatially white errors should indeed be well described by a model that is linear in the initial error amplitude $\eta$.

The numerical results (see Fig. 3) suggest furthermore that the dynamics of $V(t)$ can be separated into three different regimes. In the first, eddy viscosity will cause the spatially uncorrelated errors to decay until error growth sets in due to the linear instability and nonlinear dynamics at intermediate and large scales. During the initial regime of error decay the energy in the high-wave number part of the initially white error spectrum will be dissipated away. The smaller the structures in the error field are, the faster they will fade away.

The resulting dynamics leads to an initial decay for $V(t)$ which can be approximated at early times by just restricting the error dynamics to the heat equation:

$$\frac{\partial \varepsilon}{\partial t} = \nu \nabla^2 \varepsilon,$$

(4)
where $\nabla^2$ is the two-dimensional Laplace operator. We now assume that the initial error field $\epsilon(t=0)$ is isotropic and white for all scales larger than the (grid) scale $L_0$. The average of the square root of the initial error variance $V_0(0)$ is, according to Eqs. (2) and (3):

$$V_0(0) = \frac{2\eta}{k_{max}^2} \int_{0}^{k_{max}} k dk;$$

(5)

the subscript ‘‘d’’ stands for ‘‘decay,’’ and we use Cartesian geometry for simplicity’s sake. Here Parseval’s formula (e.g., Morse and Feshbach, 1953) was used, $k^2 = k^2_x + k^2_y$ is the wave number, and $k_{max} = \pi/L_0$ corresponds to the grid size $L_0$.

The solution of the heat equation for the initial condition $\epsilon(t=0)$ yields:

$$V_0(t) = \frac{\eta}{k_{max}^2} \int_{0}^{k_{max}} 2k \exp(-\nu k^2 t) dk = \frac{\eta}{\nu k_{max}^2} \left[1 - \exp(-\nu k_{max}^2 t)\right].$$

(6)

It is clear from Eq. (6) that each component of the initial error decays with a rate proportional to its wavelength $k$ squared. The decay of the square root of the stream-function’s relative error variance $V_d(t)$ is the weighted mean of these individual decays and thus its behavior is more complex than a simple scalar exponential law with a constant exponent (see also Fig. 3).

For the numerical model of Section 3, Eq. (6) is found to be a very good approximation for the early stages of error evolution because, for scales of the order of the grid resolution, the dynamics is indeed dominated by dissipation. We neglect damping due to bottom drag, which is independent of the length scale and thus negligible at small scales.

The second regime starts when the spatial structure of the stream-function error has shifted toward greater energy at larger wavelengths and resembles therewith the difference between two nearby OGCM solutions for the stream function $\psi(\lambda, \phi, t)$. At this point, the flow’s baroclinic and barotropic instabilities — which act at selected ranges of scales, intermediate or large — take over and induce a dominantly exponential growth of $V(t)$. This growth cannot continue indefinitely, because of the system’s forced-dissipative and quadratically nonlinear character, which limits the total energy available to the flow (Lorenz, 1963; Ghil and Childress, 1987, Chap. 5). These general considerations are supported by the numerical results on the chaotic behavior of the wind-driven ocean circulation for strong enough wind stress (Jiang et al., 1995; Berloff and Meacham, 1997). The error $V(t)$ will thus have to reach saturation, which represents the third regime.

In the second and third regime, the dynamics of the stream-function error $V(t)$ can be approximated by the solution $V_{gr}$ of the ordinary differential equation:

$$\frac{dV_{gr}}{dt} = \mu \left(V_{gr} - \frac{V_{gr}^2}{V_0}\right);$$

(7)

here $V_0$ is the relative rms stream-function difference between two independent realizations of the model solutions, and $\mu$ is the exponential growth rate of $V_{gr}(t)$ during the...
second regime. Eq. (7) was suggested by Lorenz (1982) for the evolution of the error energy in numerical weather prediction models. The solution of this equation is given by:

\[ V_g'(t) = \frac{V_e}{2} \left( 1 + \tanh \left[ \mu (t - T^*) \right] \right), \]  

(8)

where \( T^* \) is the “saturation time” determined by \( V(T^*) = V_e/2 \). The overall error evolution \( V(t) \) can then simply be approximated by the sum \( V(t) \) of the two contributions:

\[ \tilde{V}(t) = V_d(t) + V_g(t). \]  

(9)

Our simple analytical model of the error decay in the first regime contains no free parameter at all. Indeed the only parameters appearing in Eq. (6), that is \( \nu \) and \( k_{\text{max}} = \pi / L_0 \) are given for the numerical model under study. The analytical model for error growth and saturation, however, depends on the three parameters \( \mu, V_e, \) and \( T^* \) that appear in Eqs. (7) and (8). These parameters cannot be explicitly calculated from the OGCM’s parameters, and thus have to be adjusted to fit the numerical results.

The relative rms difference \( V_e \) between two independent realizations can be measured by comparing the rms difference between the stream-function field at some time \( t_0 \) and at later times \( t \). This value grows very quickly with \( t - t_0 \), up to a time scale of order \( T^* \). The time scale \( T^* \) measures the duration for which a straightforward persistence forecast is useful, and is simply estimated by:

\[ T^* = \frac{L_R}{U}. \]  

(10)

where \( U \approx 0.25 \text{ m s}^{-1} \) is the typical large-scale velocity of our model ocean and \( L_R \approx 40 \text{ km} \) (see Section 3). This estimate leads to a persistence time scale \( T^* \) on the order of a few days, which agrees quite well with the numerical results plotted in Fig. 3 (dashed curve). It can easily be seen from the same figure that \( T^* \ll T \ll T_{\text{er}} \). The value of \( V_e \) is then obtained, using the ergodicity of our system, by calculating the rms value of \( \psi'(t + t_0) - \psi(t_0) \) for different values of \( t \gg T^* \) and \( t_0 \).

The exponential error growth rate in our experiments varies significantly with the initial state \( \psi \) from which each experiment is started, but shows only a weak dependence on the perturbation \( \epsilon \) (not shown). This type of dependence of error growth on \( \psi \) but not on \( \epsilon \) has been associated with variations in the leading “local Lyapunov exponent” (Legras and Ghil, 1985). Local Lyapunov exponents (Voglis and Contopoulos, 1994) depend, however, quantitatively on the coordinate representation of the dynamical system (Elskens, 1997) and we did not feel, therefore, that the connection between \( \mu \) and these exponents was worth exploring in greater detail.

The above assumption of an initial error field that is isotropic and white-in-space is rather crude. The results shown in Figs. 3–5 demonstrate however that our analytical model captures the dynamics of the error field to a fairly high degree of accuracy. The best fit of our model to the data for \( \nu = 2.5 \cdot 10^6 \text{ cm}^2 \text{ s}^{-1} \) is obtained for \( \mu = (20
The qualitative aspects of the dynamics in the second regime (exponential growth) agree with the findings of Dalcher and Kalnay (1987) (see also Lorenz, 1982) in atmospheric predictability experiments. The saturation in the third regime, however, seems to be different. Our numerical model appears to switch to a slower growth rate between the inflection point of the error-growth curve and the subsequent saturation to \( V_e \) (Fig. 3). This might be explained by a slowly varying component of the large-scale circulation or by a switch from baroclinic to barotropic instability being the primary cause of the error growth. We have not explored this issue any further, since the primary purpose of this paper is to study the early stages of error behavior, which had been largely neglected until now.

The initial error decay described herein is not present in the findings of Dalcher and Kalnay (1987). In those experiments and similar ones that use operational results from numerical weather prediction models, the initial difference is between fields that are already in near-geostrophic balance, because of the initialization procedures used in such operational models (Daley, 1991), and have a statistical-equilibrium spectrum of initial errors that is not white (Phillips, 1986). Hence, there is very little initial error decay or none and the equivalent of our first regime is essentially absent. In studies that are not based on models using a continuous assimilation-forecast cycle, in particular for oceanic predictability experiments (e.g., Brasseur et al., 1996), the initial error decay does occur, but is less pronounced than in the results reported here because the white-noise error represents only a part of the overall difference between initial states (see Section 6 below).

Our analytical model for the initial error decay can also be extended to cases where the usual dissipation operator \( \nu \nabla^2 \) is replaced by a hyperdissipation operator \( \nu_{\text{hy}} (\nabla^2)^2 \) (commonly called hyperviscosity), although no closed-form solution for the error decay can be found in this case. Hyperdissipation damps the smallest scales even more strongly and shortens therewith the dissipation time scale — within which dissipation dominates nonlinearity — and thus leads to a shorter decay time. Hyperdissipation seems, for this reason, to be a better numerical choice in Eq. (4), although it is known to give rise to spurious oscillations (e.g., Jiménez, 1994).

6. Concluding remarks

The simplified, small-domain numerical model of Section 3 allowed us to average OGCM error evolution over a substantial ensemble of solutions and random perturbations thereof. We showed that this numerical model’s average error evolution is well
fitted by a simple analytic model with few parameters, expressed in (Eqs. (6), (7) and (9) of Section 4.

To determine the relevance of these results to full-basin OGCMs with realistic topography and wind stress, we compared them to results of two experiments using such an OGCM with the DAMEE-NAB configuration, carried out by C.-C. Ma (pers. commun., 1997). The model is version MOM1.0 of the same OGCM as in Section 3, with a 0.5° × 0.5° horizontal resolution and 15 vertical levels (Ma et al., 1999). The model domain is the North Atlantic between 6°N and 50°N, and all the surface forcing functions correspond to DAMEE-NAB specifications (Z. Sirkes, pers. commun., 1997).

This NAB model version uses hyperdissipation, and so we expect its error dynamics to enter earlier into the fast error-growth regime than in the smaller-basin MOM2.2 we studied so far. The time $T_d$ is found indeed to equal 3–9 days in the two NAB experiments, shorter than the 6-to-12 days found by our model.

Eqs. (8) and (9) show, however, that the error decay is quite sensitive to the parameters involved. This makes it difficult to quantitatively apply the present theory when the exact values of the parameters, such as $\mu$, $V$, and $T'$, have not been adjusted by numerical experiments like the ones described above. The crucial role of parameter estimation in both data assimilation and prediction is well known (Dee, 1995; Ghil, 1997; Navon, 1997; and references therein). The type of experiments presented here is one way to estimate the parameters involved in the error-evolution model of Section 3.

We have considered so far only the decay of purely white noise and calculated the corresponding time scale. This time scale is an upper bound for the decay time since the initial error field contains also spatially “colored” errors (Balgovind et al., 1983; Jiang and Ghil, 1997).

In most applications, including data assimilation, the spatially white error only represents a part of the total error. The decay of this part will be subject to the present theory, while the colored part of the error may start to grow right away, dominating the overall error after some short time. The findings of Fig. 8 in Brasseur et al. (1996) illustrate the latter statement by showing the divergence of a numerical ocean model starting from initial states that are taken from the same model’s evolution while continuously updated by satellite altimeter data. These authors compared rms differences in stream function of forecast runs started on distinct days at a later time, following the methodology of Lorenz (1982).

We expect that the white-noise part in the total error is larger when the time lag between the start of the different runs is smaller. This should lead to a longer error-decay time for smaller lag, which is indeed the case in Fig. 8 of Brasseur et al. (1996).

The main points of the present paper are that: (i) error evolution is scale-dependent (see also Lorenz, 1969; Dalcher and Kalnay, 1987), and (ii) errors may decay before starting to grow. It does, therefore, not suffice in predictability studies to determine error-doubling times from a ratio of the error at some finite time to that at initial time: it is the minimal error, possibly after initial time, that doubles. The simple scalar model of error evolution developed here can serve as guidance for the development of more sophisticated models that take into account the covariance matrix of the error structure, in its full or reduced form.


Acknowledgements

We are grateful to K. Ide and C. R. Mechoso for extensive discussions, to R. Pacanowski for his help with the MOM 2.2 implementation, and to C.-C. Ma for sharing with us results from his numerical experiments with the full-size OGCM in the DAMEE-NAB configuration. Constructive comments from two anonymous referees helped refine the error-evolution model and clarify the presentation. Our work was supported by ONR grant N00014-93-1-0673 and NASA grant NAG5-713. This is contribution no. 5075 of UCLA’s Institute of Geophysics and Planetary Physics.

References