Vertical mixing schemes in the coastal ocean: Comparison of the level 2.5 Mellor-Yamada scheme with an enhanced version of the K profile parameterization

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[1] The performance of two vertical mixing parameterizations in idealized continental shelf settings is analyzed to assess in what aspects and under what conditions they differ. The level 2.5 Mellor-Yamada turbulence closure (M-Y) is compared with an enhanced version of the K profile parameterization (KPP), which has been appended to include a representation of the bottom boundary layer. The two schemes are compared in wind-driven one- and two-dimensional shallow ocean settings to examine differences in (1) the surface boundary layer response, (2) the response when surface and bottom boundary layers are in close proximity, and (3) the response when the horizontal advective effects of a coastal upwelling circulation compete with the vertical mixing processes. The surface boundary layer experiments reveal that M-Y mixes deeper and entrains more than KPP when the pycnocline beneath the wind-mixed layer is highly stratified and mixes less when it is weaker. This is related to the role of vertical diffusion of turbulent kinetic energy in M-Y and the nature of the interior shear mixing parameterization of KPP. In shallow water when surface and bottom boundary layers impinge on each other, the stronger mixing at the interface produced by KPP can lead to much more rapid disintegration of the pycnocline. The two-dimensional upwelling circulation experiments show that the two schemes can produce quite similar or significantly different solutions in the nearshore region dependent on the initial stratification. The differences relate to the stronger suppression of turbulence by M-Y under the restratifying influence of horizontal advection of denser water in the bottom boundary layer. INDEX TERMS: 4255 Oceanography: General; Numerical modeling; 4279 Oceanography: General: Upwelling and convergences; 4568 Oceanography: Physical: Turbulence, diffusion, and mixing processes; KEYWORDS: vertical mixing parameterizations, wind-driven mixing


1. Introduction

[2] Accurate parameterization of vertical mixing processes has been a long-standing issue in ocean circulation modeling. A variety of schemes have been developed to face this challenge; however, very much is expected of them. Wind-driven turbulent boundary layer mixing, convection forced from the surface or at an overturning front, internal wave breaking and Kelvin-Helmholtz instabilities at the pycnocline are several of the typically unresolved mechanisms which contribute significantly to the vertical redistribution of both momentum and scalars. Numerical models focused on the mesoscale offer spatial information on scales of a kilometer in the horizontal and 1–10 m in the vertical. The fields at these resolutions may correlate strongly, weakly or not at all with the dynamics of small-scale processes. Nonetheless, vertical mixing parameterizations based on the values of the variables at the resolved scales have been shown to represent some mixing processes rather well and improve the accuracy of circulation models in general. As the demand for more accurate, higher-resolution real ocean simulations increases it is important to continue to assess the performance of currently available parameterizations to determine what can be done to improve their quality.

[3] Numerous comparisons between vertical mixing parameterizations have been undertaken [Price et al., 1986; Large et al., 1994; Kantha and Clayson, 1994; Large and Gent, 1999; Burchard and Bolding, 2001; Wijesekera et al., 2003]. Often they focus on open ocean settings because
the significant level of horizontal homogeneity these environments offer facilitates model-data comparisons by alleviating the need to consider lateral advective effects. It is arguably in these settings that the models have achieved their greatest success; however, for general use the parameterizations must perform reasonably in a much broader range of settings. A particularly challenging environment for comparison of vertical mixing parameterizations is that of the coastal ocean. There the water column can range from well-mixed surface-to-bottom to highly stratified over a few kilometers and the strong horizontal velocities and shallowness of the water column can lead to interaction between surface-forced and bottom-forced boundary layers which come in close proximity to each other. In such complex environments, where the performance of the vertical mixing parameterization is most dubious, comparison with observations is most difficult to achieve. Coastal circulation models have improved to the point that they can capture many of the features of mesoscale coastal processes but rarely with the accuracy necessary for vertical mixing estimates to be directly compared with small-scale turbulent measurements in the field.

In lieu of model-data comparison a great deal can still be learned about the relative performance of vertical mixing parameterizations through sensitivity tests in idealized coastal ocean settings. By developing a detailed understanding of why parameterizations give different solutions one can better determine what attributes are critical to obtaining a certain response. Although this paper does not promote one parameterization as superior, by clarifying the nature and causes for differences, it can suggest how future theoretical, laboratory, high-resolution numerical or field work can most profitably advance our understanding of vertical mixing processes and lead to improvement in parameterizations.

The approach of this study is to compare two mixing schemes in a series of idealized coastal ocean settings. The study is limited to purely wind driven systems to isolate specific aspects of their response. Three characteristics of the coastal ocean are considered specifically: (1) the potential for the stratification to range over several orders of magnitude, (2) the potential for surface/bottom turbulent boundary layers to interact in shallow water, and (3) the potential for horizontal density gradients to interact with vertical mixing in association with such phenomena as coastal upwelling fronts.

The two parameterizations that will be considered are the K profile parameterization of Large et al. [1994] (referred to as KPP) and the Mellor and Yamada level 2.5 closure scheme Mellor and Yamada [1982] (referred to as M-Y). Whereas the Mellor-Yamada scheme has become something of the “industry standard” for coastal ocean application, the KPP scheme has gained respect as an alternative in deep ocean applications and has been shown to compare favorably to M-Y in such situations [Large and Gent, 1999; Large et al., 1994]. There are numerous other parameterizations that could be included in a sensitivity study such as this [Canuto et al., 2001; Burchard and Baumert, 1995; Price et al., 1986; Pacanowski and Philander, 1981]. These two were chosen because they represent popular members of two different classes of parameterization. The estimates of vertical mixing coefficients made by the KPP scheme are based on the surface boundary forcing and the state of the resolved velocity and potential density fields instantaneously in a vertical column of water. M-Y, on the other hand, considers the energetics of the mixing explicitly by solving prognostic equations for turbulent kinetic energy and length scale. In doing so, the mixing estimates carry information about the time history of the flow and can effectively both advect and diffuse.

The KPP scheme was primarily designed as an advanced surface boundary layer approximation coupled at its base with simple parameterizations to represent a range of mixing processes in the ocean interior. Consequently, it takes no special consideration for the presence of a lower boundary and can produce erroneous mixing as a result in shallow water. As part of this study, the original scheme is appended to include a representation for the turbulent bottom boundary layer. This improvement gives the model general applicability to continental shelf and estuarine flows.

Three model setups will be discussed. These are (1) a simple one-dimensional wind driven surface boundary layer over a range of stratifications, (2) interacting surface and bottom boundary layers in a one-dimensional setting over a range of stratification intensities and water depths, and (3) a two-dimensional coastal upwelling on a gently sloping continental shelf.

After a discussion of the vertical mixing parameterizations in the next section the circulation model and the three simulation setups will be presented. This will be followed by results and analysis of each case and a final discussion.

2. Vertical Mixing Parameterizations

In the published literature one can find a variety of alterations, adjustments and “corrections” associated with the parameterization being examined here. Thus it is important to present some details of their specific implementations in this study as some results may depend on the particulars of these formulations. A variety of alternatives are offered for coefficient values and functional representations, particularly for the Mellor-Yamada scheme [Mellor and Yamada, 1974, 1982; Galperin et al., 1988; Blumberg et al., 1992; Kantha and Clayson, 1994; Mellor, 2001], but also for KPP [Large and Gent, 1999]. In this section the addition of a bottom boundary layer to the KPP scheme will also be discussed.

2.1. Mellor-Yamada Parameterization

The formulation for the Mellor-Yamada level 2.5 closure presented here is based on Mellor and Yamada [1982] with the modifications suggested by Galperin et al. [1988] and the parameter adjustments presented by Kantha and Clayson [1994]. Coefficients for vertical eddy viscosity and diffusivity are estimated as

\[ K_v = q\tilde{\nu}S_M \]  
\[ K_p = q\tilde{\nu}S_H \]

where \(0.5q\tilde{\nu}^2\) is the turbulent kinetic energy, \(\tilde{\nu}\) is a limited turbulent length scale, \(\rho\) denotes potential density, and \(S_M\)
and $S_H$ are stability functions for momentum and scalars. $q^2$ and $q^2 l$ are calculated prognostically through the following equations (written in $z$ coordinates for simplicity here),

$$\frac{\partial q^2}{\partial t} + v \cdot \nabla q^2 = 2K_q \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right) + \frac{2g}{\rho_o} K_q \frac{\partial q}{\partial z} + 2g^3 B_l^2 + \frac{\partial}{\partial z} \left[ K_q \frac{\partial q^2}{\partial z} \right]$$

(3)

$$\frac{\partial q^2 l}{\partial t} + v \cdot \nabla q l = E_l \left[ K_q \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right) + \frac{g}{\rho_o} K_q \frac{\partial q}{\partial z} \right] - \frac{q^2}{B_l} W + \frac{\partial}{\partial z} \left[ K_q \frac{\partial q^2 l}{\partial z} \right]$$

(4)

[12] In these equations $K_q$ is estimated as $0.41K_v$. $W$ is a wall proximity function expressed as,

$$W = 1 + \frac{E_l}{\nu^2} \left[ \frac{1}{v_1 - z} + \frac{1}{H + z} \right]$$

(5)

The stability functions in the expressions for eddy viscosity and diffusivity are

$$S_H = \frac{A_1 \left( 1 - 6A_1 B_1^{-1} \right)}{1 - \left( 3A_2 B_2 \left( 1 - C_1 \right) + 18A_1 A_2 \right) G_{H_H}}$$

(6)

$$S_M = \frac{A_1 \left( 1 - 3C_1 - 6A_1 B_1^{-1} \right) - S_H \left[ G_{H_H} \left( 18A_1^2 + 9A_1 A_2 \left( 1 - C_2 \right) \right) \right]}{1 - 9A_1 A_2 G_{H_H}}$$

(7)

Parameters in the above equations are set as

$$A_1 = 0.92, A_2 = 0.74,$$

(8)

$$B_1 = 16.6, B_2 = 10.1,$$

(9)

$$C_1 = 0.08, C_2 = 0.7, C_3 = 0.2,$$

(10)

$$E_1 = 1.8, E_2 = 1.33$$

(11)

and

$$G_{H_H} = \min \left( \frac{n^2}{q^2}, 0.028 \right).$$

(12)

[13] The turbulent length scale contained in the $q^2 l$ equation differs from $l$ which is used in the estimates for $K_v$ and $K_p$. As suggested by Galperin et al. [1988] a maximum is placed on the length scale of eddies that contribute to vertical mixing in stably stratified conditions. This limit is

$$l = \min \left( l, \frac{0.53d}{N} \right).$$

(13)

where $N$ is the buoyancy frequency.

[14] The total vertical mixing coefficients are the sum of the turbulent contribution plus a constant “background” level. For these experiments the background coefficients are set as $\nu_M = 10^{-7} \text{ m}^2 \text{ s}^{-1}$, $\nu_H = 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and $\nu_q = 10^{-8} \text{ m}^2 \text{ s}^{-1}$.

[15] A third-order upwind scheme is utilized to time step the advection terms [Shchepetkin and McWilliams, 1998]. The diffusion terms are time stepped semi-implicitly using a Crank-Nicholson formulation. The turbulent dissipation terms in both the $q^2$ and $q^2 l$ equations are discretized in time following the implementation in POM [Blumberg and Mellor, 1987]. The $q^2 l$ in this term is expressed as the product of $q_l^n$ (the value calculated at the previous time step $n$) multiplied by $q^2 l^{n+1}$ (the current value being estimated at time step $n + 1$). Because this term involves “future” values of the variable it also must be solved for implicitly.

[16] The turbulent production terms involve estimates of the square of the vertical shear in the horizontal velocity and the vertical potential density gradient. It is often necessary to apply Shapiro filters horizontally to these fields to avoid extremely noisy results. This approach is taken here for the two-dimensional simulations.

[17] As a final note on the M-Y formulation, it is worth pointing out two features of the parameterization that will play an important role in this study. (1) Here $q^2$ and $q^2 l$ diffuse vertically such that mixing can exist above background levels in portions of the water column where the mean shear and stratification do not support production of turbulence locally. (2) Buoyant suppression exceeds shear production of turbulence in a stably stratified fluid when the gradient Richardson number ($R_{Ig}$) exceeds approximately 0.21, where

$$R_{Ig} = \frac{N^2}{Sh^2}$$

(14)

in which $Sh^2$ is the square of the vertical shear in the horizontal velocity.

2.2. Large, McWilliams, and Doney Parameterization

2.2.1. Basic Formulation

[18] A description of the KPP formulation as it applies to this study will be discussed next. Several aspects of the full formulation which are not relevant to this study, will be excluded. These include the counter-gradient flux term and other portions of the boundary layer formulation associated with nonneutral surface buoyancy fluxes. The double-diffusive mixing parameterization for the interior is also excluded.

[19] The K profile parameterization of Large et al. [1994] matches separate parameterizations for vertical mixing of the surface boundary layer and the ocean interior. A formulation based on boundary layer similarity theory is applied in the water column above a calculated depth ($h_{ssl}$). This is matched at the base of the boundary layer with mixing formulations to account for local shear and internal wave effects.

[20] Viscosity and diffusivities at model levels above $h_{ssl}$ are expressed as the product of the length scale $h_{ssl}$, a turbulent velocity scale $w_t$, and a nondimensional shape function $G_f$.

$$K_q = h_{ssl} w_f(\sigma) G_f(\sigma)$$

(15)
where \( \sigma \) is a nondimensional vertical coordinate ranging from 0 at the surface to 1 at the base of the surface boundary layer (at \( h_{slb} \)). The subscript \( f \) refers to either momentum or potential density in this study.

[21] Under neutral surface forcing conditions (no heat or salinity fluxes), \( h_{slb} \) is calculated as the minimum of the Ekman depth, estimated as,

\[
h_e = 0.7u_f/f
\]  

(16)

(where \( f \) is the Coriolis parameter and \( u_f \) is the bottom friction velocity) and the shallowest depth at which a critical bulk Richardson number (\( R_i_b \)) is reached (set here to 0.3). The bulk Richardson number, \( R_i_b \), is calculated as

\[
R_i_b(z) = \frac{(B_e - B(d))d}{| V_r - V(d) |^2 + V_q^2 |d|}
\]  

(17)

where \( B \) is the buoyancy, \( V \) is horizontal velocity and \( d \) is distance from the surface. The \( r \) subscript refers to the value the field has at a near-surface reference depth, which here is specified as the top model grid level. \( V_q \) is an estimate of the turbulent velocity contribution to velocity shear and is calculated as

\[
V_q^2 (d) = \frac{C_v (-\beta_c) ^{1/2}}{R_i_b} \left( c_s e^{-1/2} d \right)^{1/2} dN_{w_f},
\]  

(18)

where \( C_v = 1.6, \ c_s = -98.96, \ \epsilon = 0.1 \) and \( \beta_c = 0.2. \)

[22] To estimate \( w_f \) throughout the boundary layer, surface layer similarity theory is utilized. Following an argument by Troen and Mahrt [1986], Large et al. [1994] estimate the velocity scale as

\[
w_f = \frac{\kappa u_f *}{\partial_y (\zeta)}
\]  

(19)

where \( \phi_f \) is a nondimensional flow profile associated with the stability parameter \( \zeta \), which varies on the basis of the stability of the boundary layer forcing. In the neutral forcing case it is identically 1 and \( \phi_f = \kappa u_f * \).

[23] The nondimensional shape function \( G(\sigma) \) is a third-order polynomial with coefficients chosen to match the interior viscosity at the bottom of the boundary layer and Monin-Obukov similarity theory approaching the surface. This function is defined as

\[
G(\sigma) = a_0 + a_1 \sigma + a_2 \sigma^2 + a_3 \sigma^3
\]  

(20)

with coefficients \( a_0 \) and \( a_1 \) specified to match boundary conditions at the surface and \( a_2 \) and \( a_3 \) determined to smoothly blend mixing within the boundary layer with the interior:

\[
a_0 = 0
\]  

(21)

\[
a_1 = 1
\]  

(22)

\[
a_2 = -2 + 3 \frac{v_r(h_{slb})}{h_{slb}w_f(1)} + \frac{\partial_y v_r(h_{slb})}{w_f(1)} - \frac{v_r(h_{slb}) \partial_y h_{slb} w_f(1)}{h_{slb} w_f^2(1)}
\]  

(23)

\[
a_3 = 1 - 2 \frac{v_r(h_{slb})}{h_{slb}w_f(1)} - \frac{\partial_y v_r(h_{slb})}{w_f(1)} - \frac{v_r(h_{slb}) \partial_y h_{slb} w_f(1)}{h_{slb} w_f^2(1)}.
\]  

(24)

Here, \( v_r(h_{slb}) \) is the viscosity calculated by the interior parameterization at the boundary layer depth. (Note that \( v \) will be used to refer to an estimate by the interior parameterization and \( K_v \) will refer to one associated with the boundary layer estimate.)

[24] The interior scheme of KPP gives estimates of the viscosity coefficient by adding the effects of shear mixing and internal wave-generated mixing (along with double-diffusive mixing, which is set to zero here). The shear mixing term is calculated using a gradient Richardson number formulation with viscosity estimated as:

\[
\nu^{sh} = \begin{cases}
\nu_0 & R_i_b < 0, \\
\nu_0 \left[1 - \left(R_i_b/R_i_0 \right)^2 \right]^3 & 0 < R_i_b < R_i_0, \\
0 & R_i_b > R_i_0.
\end{cases}
\]  

(25)

where \( \nu_0 = 5.0 \times 10^{-3} \ m^2 \cdot s^{-1}, \ R_i_0 = 0.7 \) and \( R_i_b \) is as defined in equation (14). The turbulent Prandtl number is assumed to equal 1 for this parameterization so \( \nu_f = \nu_{vf}. \)

[25] Internal wave-generated mixing serves as the background mixing in the KPP scheme. It is specified as a uniform value for each scalar and momentum. The values suggested by Large et al. [1994] for eddy diffusivity due to internal wave activity is \( 1.0 \times 10^{-5} \ m^2 \cdot s^{-1}. \) It is based on the deep ocean data of Ledwell et al. [1993]. The internal wave generated mixing of momentum is specified as ten times this value following Peters et al. [1988]. These values are found to lead to unrealistically rapid broadening of the pycnocline under reasonable forcing conditions in highly stratified coastal waters. Therefore the background values for viscosity and diffusivity in this study are reduced to values identical to those used for the Mellor-Yamada scheme \( (1.0 \times 10^{-5} \ m^2 \cdot s^{-1} \ for \ momentum \ and \ 1.0 \times 10^{-6} \ m^2 \cdot s^{-1} \ for \ potential \ density). \) The contribution of internal wave generated mixing to continental shelf circulation remains an important research question.

### 2.2.2. Enhancement of the KPP Scheme: Appendin g a Bottom Boundary Layer Parameterization

[26] The KPP scheme works by matching vertical mixing rates appropriate for the surface boundary layer with ones appropriate for the interior of the flow. Thus the boundary layer is primarily determined by the surface fluxes but is also influenced by the interior. The turbulent velocity scale is solely determined by the surface forcing. The boundary layer depth is determined either by property changes relative to the surface layer or by an estimate of the Ekman layer thickness. However, the shape function that determines the profile of the mixing coefficient over the boundary layer is explicitly dependent on the vertical mixing of the interior. The value and gradient of the mixing predicted by the interior scheme at the boundary layer depth have a significant impact on mixing throughout the boundary layer. Large et al. [1994] justify this by referring to atmospheric boundary layer work by Kurzeja et al. [1991] and Kim and Mahrt [1992].

[27] Although the interior mixing mechanisms Large et al. [1994] suggested may be quite appropriate for the deep ocean, they fail to appropriately represent mixing near the bottom boundary and, consequently, are inadequate for application on a shallow continental shelf. When an
adequately strong velocity shear exists near the bottom boundary, the gradient Richardson number-based, shear-generated mixing term of KPP will predict high levels of mixing. However, the formulation does not associate mixing intensity with a length scale to account for proximity to the boundary. If the bottom sheared flow is isolated from the surface boundary layer, a region of intense mixing will form which disobeys boundary layer similarity theory. If the surface boundary layer extends near to the bottom of the domain, as it can in shallow water, both the magnitude and the gradient of the mixing coefficients used to estimate the shape function throughout the water column will be unrealistic. Figure 1 shows several profiles of vertical viscosity over a gradually sloping continental shelf during an upwelling simulation using the original KPP scheme and the modified scheme to be discussed below. The viscosity profiles with the original scheme show gross overestimates in the well-mixed water column nearshore and unreasonable representation of the bottom boundary layer across the shelf. These problems arise primarily because the estimate of $\nu$ at the bottom interior grid point ($k = 1$) approaches the maximum allowable by the shear generated mixing scheme $0.005 \text{ m}^2 \text{s}^{-1}$, while the mixing coefficient at the ocean floor is set to zero (leading to large $\partial_z \nu$ there).

To alleviate this situation, we append a bottom boundary layer approximation following the KPP surface boundary layer representation. As was the case for the surface boundary layer we determine a mixing profile for the bottom boundary layer constrained by a requirement of matching Monin-Obukov similarity scaling as the boundary is approached. Thus viscosity in the bottom constant stress layer should reduce to

$$K = \kappa u^*_b z,$$

where $u^*_b$ is the bottom friction velocity. Only the case of zero bottom buoyancy flux is considered here as this is usually appropriate. The turbulent velocity scale for the bottom boundary layer reduces to $w^*_b = \kappa u^*_b$. The bottom boundary layer depth is determined as it is for the surface layer using an Ekman layer depth estimate and a bulk Richardson number criteria. The bottom friction velocity and reference velocity and buoyancy fields are obtained using the model bottom $u$, $v$ and $\rho$ grid points.

The bottom boundary layer estimate can connect with the surface and interior parameterizations in three ways.

1. If the bottom boundary layer does not extend into the surface boundary layer then the bbl parameterization simply matches with the interior just as the surface boundary layer scheme does.

2. If it extends over the entire depth of the water column (or the surface boundary layer does), the shape function is specified to properly match with neutral law-of-the-wall behavior ($\partial_z K_v = \kappa u^*$), at the top and bottom boundaries.

3. When surface and bottom boundary layers intersect but do not fully overlap, vertical mixing due to the effects of each must be matched. Vertical mixing in the

![Figure 1](image-url). Profiles of vertical viscosity coefficient at day 2 of upwelling simulations with the original KPP scheme and a version that has been appended to include a representation of the bottom boundary layer. Density contours are also drawn to delineate the pycnocline.
3. Model Setup

The two vertical mixing schemes are compared as implemented in ROMS [Haidvogel et al., 2000], a hydrostatic, primitive equation, generalized sigma coordinate model. For these experiments a single prognostic equation for potential density is used rather than separate equations for temperature and salinity. A third-order upwind scheme is used for horizontal advection and a fourth-ordered-centered scheme is used for vertical advection of all fields. A small amount of “horizontal” Laplacian-diffusion (along sigma levels) is also utilized (2.0 m$^2$ s$^{-1}$) for momentum and potential density. Three model setups are explored which emphasize different aspects of vertical mixing in the coastal ocean.

3.1. Case 1: Surface Boundary Layer Response to a Wind Stress

The first setup examines the surface boundary layer response to a wind deepening event (at midlatitude). Model simulations are performed in a one-dimensional, 20 m deep domain with uniform 0.5 m vertical resolution. A moderate wind stress is applied in the y direction which is spun up over the first inertial period of the simulation. It is sustained for 1.6 days then ramped back down to zero by day 3 as displayed in Figure 2. A wind “pulse” experiment is also briefly examined (Figure 2). It has the same integrated wind stress over the three day period but is applied as a forcing 3.33 times as strong for one-third the duration. The bottom stress is set to zero in these simulations to isolate the surface boundary layer response as if in an infinitely deep ocean.

The stratification on a continental shelf can vary completely well mixed because of convective cooling or during strong wind events to very highly stratified because of surface heating and/or riverine input of fresh water. Here we want to compare the surface boundary layer response over a broad range of stratification. The water column is initially at rest with no stratification in the top 7.5 m and uniform stratification beneath this depth. Four initializations are considered in which the interior stratification is varied. The highest stratification, $N^2 = 0.0098$ s$^{-2}$ (or 1 kg/m$^3$/m) we label $N_o$. The lower three are $N_o/10$, $N_o/100$ and $N_o/500$.

3.2. Case 2: Surface/Bottom Boundary Layer Interaction

The second setting again uses a one-dimensional, wind-forced, stratified water column. Here the net transport of mass over the water column in the across-wind direction is forced to equal zero. This specification drives the development of bottom currents opposing the surface Ekman flux. In shallow enough water a bottom boundary layer forms which may interact with the surface wind forced one. In many ways this setup is analogous to the circulation that develops in two-dimensional upwelling in which an alongshore wind drives offshore transport which is balanced onshore flow in a bottom boundary layer.

A moderate wind stress is spun up over the first inertial period as in the case 1 experiment, but here it is sustained until the end of the experiment at day 6. Bottom stress is calculated following a quadratic drag law as,

$$ (\tau_b, \tau_v) = \rho_b C_d \left( \sqrt{u_b^2 + v_b^2} \right) (u_b, v_b) \quad (27) $$

where $\tau_b$ denotes bottom stress, $u_b$ and $v_b$ are model velocities components at the bottom grid point and $C_d$ is a drag coefficient specified as,

$$ C_d = \kappa^2 \left( \ln \frac{z_b}{z_o} \right)^{-2} \quad (28) $$

where $\kappa$ is vonKarman’s constant, $z_b$ is the distance the bottom $u$ or $v$ grid point is from the seafloor and $z_o$ is a roughness height specified as 1 cm.

The density field is initialized again with a 7.5 m thick well-mixed surface layer above a uniformly stratified interior. Initial stratification of the pycnocline is varied from 0.02 kg/m$^3$/m to 1 kg/m$^3$/m ($N_o/50$, $N_o/10$ and $N_o$) in sensitivity studies. To vary the degree of interaction.
between the surface and bottom boundary layers that form, the water depth is varied between 15 and 20 m.

3.3. Case 3: Vertical Mixing With Advective Processes, Two-Dimensional Coastal Upwelling

[39] Cases 1 and 2 explore the dynamics of a stratified water column under the influence of vertical mixing alone. In case 3 we explore the interplay of mixing with advective processes in a simple two-dimensional coastal upwelling setting. The domain is set up with a 6-m deep coastal wall at the western boundary and a radiating offshore open boundary 100 km to the east in 106 m of water (bottom slope of 1 m km\(^{-1}\)). This represents a broad, shallow continental shelf analogous to that found off the coast of New Jersey. (A comparison of mixing schemes under upwelling and downwelling forcing on a significantly steeper slope, characteristic of the Oregon shelf, has been undertaken by Wijesekera et al. [2003].) The shallow bathymetry examined here focuses attention on the upwelling evolution in water less than 25 m where the circulation is likely most sensitive to the vertical mixing processes.

[40] Horizontal resolution is varied from roughly 400 m in the nearshore region to 4 km offshore. Forty vertical levels are utilized. Wind forcing is identical to that in the case 2 experiment and bottom stress is again specified using a quadratic drag law (equation (27)). A free-slip condition is applied at the coastal boundary.

[41] Sensitivity tests were performed with horizontally uniform initial stratification over the full range discussed in the previous 2 cases. The results and discussion below will focus primarily on results from a simulation initialized with a 5 m thick pycnocline at \(N_o/10\) stratification located between 7.5 and 12 m depth between well mixed surface and bottom layers.

4. Single Boundary Layer Response: Case 1

[42] Entrainment into the surface boundary layer provides a basic measure of how the two mixing parameterizations perform differently. While the results and discussion below focus on only two idealized forcing conditions the qualitative differences examined here hold in general for wind-forced single-boundary layer simulations.

[43] Figure 3 shows a time series of change in surface density at four different initial stratifications for the two mixing schemes. \(N_o^2 = 0.0098\) s\(^{-1}\).

![Figure 3](image_url)

**Figure 3.** Time series of change in surface density at four different initial stratifications for the two mixing schemes. \(N_o^2 = 0.0098\) s\(^{-1}\).
more than the other. To explain how the differences develop, we first examine the basic dynamics of this system. Figure 4 displays contour plots of the potential density anomaly and the velocity field in the direction normal to the wind stress for the No/10 case. It also shows instantaneous profiles of the two fields at day 2.4. The density anomaly contours show the pycnocline deepening and intensifying shortly after the onset of the wind event. Inertial oscillations develop in the velocity field as a result of the finite duration over which the winds were spun-up. In accordance with the results from Figure 3 the pycnocline in the M-Y simulation is observed to deepen continually from the beginning to the end of the wind event. In the KPP simulation the deepening ceases nearly completely by day 1.5. The profile of density at day 2.4 shows that the KPP produces a shallower pycnocline which starts more abruptly at the base of the surface boundary layer. The M-Y profile displays a more gradual increase in density with depth. This results in the much sharper changes in $u$ component velocity at the base of the boundary layer with KPP observed in the lower panels. Momentum is more well mixed with KPP resulting in lower maximum surface velocities despite the fact that the mixed layer it produces is shallower than that with M-Y.

The shear mixing formulation of the KPP scheme depends explicitly on the gradient Richardson number for estimating the viscosity and diffusivity coefficients. Recalling the shear mixing formulation from equation (25), turbulent mixing is initiated at $Ri_g < 0.21$. To understand how the mixing schemes produce the differences mentioned above it is useful to examine the evolution of $N^2$, $Sh^2$ and $Ri_g$ over the wind event. Figure 5 depicts time versus depth contour plots of these fields for the two schemes for the No/10 case. The KPP produces a significantly intensified pycnocline by day 1.2. This results not only from the direct effect of the wind stress but also from the shear at the base of the boundary layer that develops intermittently. After the initial deepening, the intense pycnocline that develops at approximately 8.5 m remains at that depth for the duration of the wind event. $Sh^2$ intermittently increases and decreases at this depth at the inertial frequency. $Ri_g$ which is the quotient of these two terms oscillates out of phase with $Sh^2$ reaching values as low as 0.5 at 8.5 m depth. The vertical gradient in $Ri_g$ remains intense at the base of the boundary layer throughout the simulation.

The intensity of the pycnocline with the M-Y parameterization does not reach as high a value as it does in KPP until approximately 8 hours later. $Sh^2$ never intensifies with M-Y to the extent it does with KPP. Consequently, the gradient Richardson number changes gradually between 6 and 9 m depth and remains well above the value of 0.21 below which shear production would produce mixing with this formulation. Despite this the boundary layer continues to deepen and the rate of entrainment into the surface boundary layer persists.

The vertical profiles of turbulent mixing coefficient $K_v$ (Figure 6) are consistent with these profiles of gradient Richardson number. $K_v$ is smaller within the boundary layer with M-Y and tapers off more gradually as it impinges on the stratification at the pycnocline. The KPP mixing coefficient remains nearly constant at all depths after day 1.2 while
turbulence continues to extend downward for the duration of the wind event with M-Y. Upon cessation of the surface stress, both schemes display a rapid drop off in turbulent mixing.

The time rate of change of potential density in these one-dimensional, horizontally homogenous simulations is equal to the vertical diffusion term,

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial z} \bar{K}_\rho \frac{\partial \rho}{\partial z}$$

Time versus depth contour plots of this term show how the two schemes deepen the boundary layer and entrain differently (Figure 7). Yellow and red areas in these figures indicate regions where the vertical flux of denser water from the pycnocline is causing the density to rise. Blue regions in these figures occur where the density is locally decreasing because of the deepening of the pycnocline. These are depths which are transitioning from being at the top of the pycnocline to the base of a relatively well mixed boundary layer. As suggested by the earlier figures, the KPP scheme mixes intensely during the first half day of wind forcing but shuts down abruptly at approximately day 1.2. The region of negative flux divergence becomes increasingly thin as the mixing weakens. The Mellor-Yamada parameterization produces periods of enhanced flux of denser water into the boundary layer coinciding with deepening events at approximately an inertial frequency. These events happen concurrently with moderate increases in $\bar{S}h^2$ and $N^2$ in Figures 5 and 6. While the region of negative flux divergence thins with each pulse in entrainment, unlike KPP, subsequently it broadens.

This suggests that the difference in the response of the two schemes rests in the interplay between the deepening of the boundary layer and the diffusion of the pycnocline upward. The vertical mixing term in the density equation can be separated into two parts.

$$\frac{\partial}{\partial z} \bar{K}_\rho \frac{\partial \rho}{\partial z} + \bar{K}_\rho \frac{\partial^2 \rho}{\partial z^2}$$

At the top of the pycnocline, $\frac{\partial \rho}{\partial z}$ is negative, $\frac{\partial^2 \rho}{\partial z^2}$ is positive and $\bar{K}_\rho$ is positive. Thus the first term on the right-hand side of this equation is negative and associated with the decrease in density due to boundary layer deepening. It acts effectively in the up-gradient direction. The second term acts in the opposite sense and represents the typical “diffusive” effect of spreading a scalar from high concentration to low. The KPP simulations are an example of a case in which these mixing terms come to nearly cancel each other at the top of the pycnocline. Figure 8 displays time versus depth contours of $\frac{\partial \rho}{\partial z}$, $\frac{\partial^2 \rho}{\partial z^2}$ and $\frac{\partial}{\partial z} \bar{K}_\rho \frac{\partial \rho}{\partial z}$ for the $N_{0}/10$ case. The bottom pair of panels shows the two component terms of the vertical mixing. ($\frac{\partial}{\partial z} \bar{K}_\rho \frac{\partial \rho}{\partial z}$ is contoured in blue, $\bar{K}_\rho \frac{\partial^2 \rho}{\partial z^2}$ is contoured in red at the same contouring interval.) Where contours overlie precisely, the two effects cancel, where contours lie “outside” of contours of the other color, that effect (deepening of the boundary layer [blue] or spreading of the pycnocline [red]) dominates. Contours are only drawn over a limited range to emphasize the areas where the differences between the schemes occur.

Figure 8 helps explain the sudden change in entrainment rate observed in Figure 3. Strong mixing at the onset of the wind event erodes the stratification while pushing the pycnocline downward. As the boundary layer deepens the
gradient and curvature in the density field below it increase. Concurrently, the shear at the base of the boundary layer also increases. A sharp gradient in $R_i^g$ is established, which with KPP promotes a very strong variation in $K_r$ and $K_v$. The stratification below the boundary layer is initially uniform, so the farther the boundary layer deepens the more intense the gradient and curvature in the density field can become. The shear, however, cannot intensify to the same degree. The maximum magnitude it may have is set by the magnitude of the wind stress. Consequently, with a steady wind forcing, once the boundary layer deepens to a point at which the shear cannot compete with the intensified stratification, the gradient Richardson number at the boundary layer base remains high and deepening ceases. There is a feedback such that, the stronger the gradient in mixing coefficient at the base of the boundary layer is, the sharper the gradient and curvature in density become. These produce a steep gradient in $R_i^g$ which in turn establishes an even steeper $\partial_z K$. This results in the two components of the vertical mixing term nearly cancelling each other out.

Turning now to the contour plots on the right-hand side of Figure 8, an explanation can be found as to why a similar shut down in mixing does not occur with the M-Y parameterization. In general, the weaker vertical gradients in $K_r$ still lead to intensified gradients and curvature in the density field at the boundary layer base. In response to this, $\partial_z K_v$ does still intensify in a manner quite similar to that found in KPP. It appears, however, that each time the gradients at the base of the boundary layer intensify, something happens shortly thereafter to promote further deepening and a new pulse of entrainment. Looking carefully at the bottom right-hand panel of Figure 8 it can be observed that each time the red and blue contours intensify and become aligned, a blue contour spreads downward farther into the pycnocline breaking the balance. As can be observed from Figure 5, there is not adequate shear at

![Figure 6](image-url)  
**Figure 6.** Time-depth contour plots of vertical viscosity in the top 12 m of the water column for the two schemes along with profiles at day 2.4.

![Figure 7](image-url)  
**Figure 7.** Time-depth contour plots of the vertical mixing term of the potential density equation. Only the top 12 m of the water column are shown.
this depth to overcome the stratification and lead to mechanical production of turbulent kinetic energy. One must conclude that the continued deepening of the boundary layer is driven primarily by diffusion of turbulence into the pycnocline. TKE accumulates with an intensified gradient at the base of the boundary layer whenever the deepening is halted. This enhanced gradient leads to enhanced downward diffusion. This allows $\frac{\partial}{\partial z}K_r$ to become nonzero deeper in the pycnocline which can sustain the entrainment.

The importance of diffusion of turbulent kinetic energy in the Mellor-Yamada scheme can be demonstrated by repeating these surface boundary layer experiments with the M-Y parameterization but with $K_Q$ set to zero. Figure 9 shows time series of near-surface density for the KPP scheme, the regular M-Y scheme and M-Y with no diffusion of TKE. The impact of omitting this term is particularly apparent with a strongly stratified pycnocline. For the No case, entrainment with the M-Y parameterizations without TKE diffusion is nearly identical to that with KPP and much weaker than the regular M-Y scheme. With $N_o/10$ stratification it is also greatly reduced though it is apparent that other effects play some role.

The above arguments explain why at higher stratification the Mellor-Yamada scheme entrains more than the K profile parameterization. The remaining issue to explain is why KPP entrains more rapidly with a weakly stratified pycnocline. The answer rests in the strength of the mixing response KPP gives when the Richardson number is low. The initial deepening of the boundary layer arises as a result of shear mixing at the pycnocline. The KPP scheme responds with an instantaneous onset of mixing whenever the gradient Richardson number drops below 0.7. The Mellor-Yamada scheme produces a gradual increase in TKE due to shear production only when $Ri_g < 0.21$. The more rapid entrainment of KPP can be observed in Figure 10, which shows the time rate of change of the density field for the $N_o/100$ case. Only the first 1.5 days of the simulation are displayed. The lower panels show the two components of the mixing term contoured at identical levels as in Figure 8. It is clear that the stronger mixing which occurs with KPP between day 0.3 and 0.7 results from the steep gradient in $\frac{\partial}{\partial z}K_r$ that the scheme produces at the boundary layer base. Recall that, during this deepening phase, stratification has not yet intensified to such an extent that shear cannot lower the Richardson number to promote mixing.

The staircase-like pattern observed in these figures and earlier ones is a result of the finite difference formulation. Shears and stratification are linearly estimated between the grid points, but $Ri_g$ and the turbulent mixing coefficients are nonlinearly related to them. As the pycnocline is pressed downward, the discretization error in the estimate of these quantities oscillates, resulting in what appear to be sudden jumps by vertical intervals equal to the grid resolution.

The strong vertical gradients in the turbulent viscosity and diffusivity coefficients that the KPP produces can both lead to enhanced and reduced entrainment relative to the Mellor-Yamada scheme. If there is adequate shear at the base of the boundary layer to reduce the gradient Richardson number below $Ri_g$, deepening can proceed. However, once stratification has intensified to the extent that shear mixing at this depth is largely shut down, entrainment with KPP diminishes sharply.
Of course, stratification is not the only factor which determines which scheme entrains more even in simulations as idealized as these. If the same integrated wind stress is applied as a strong pulse lasting one-third the duration of the “steady” forcing examined above, the results differ. Figure 11 shows time series of surface density for simulations with stratifications of $N_0$, $N_0/10$ and $N_0/100$. While for the strongly stratified pycnocline M-Y still entrains more, for both $N_0/10$ and $N_0/100$ the reverse is true. The stronger wind stress in these simulations promotes stronger shear at the base of the surface boundary layer. The strong response of the shear mixing formulation of KPP exploits this more effectively for weaker stratification. The initial boundary layer deepening phase lasts for the duration of the wind event and KPP entrains more.

The differences between the results given by these two vertical mixing parameterization in reality come from both algorithmic and numerical sources. The above section discussed in detail how differences in the formulation of the two schemes led to different results in the simulations. Here we briefly mention how the parameterizations are sensitive to some aspects of the numerics. In particular sensitivity to vertical resolution is examined.

The $N_0/10$ experiment discussed in detail above, with a grid resolution of 0.5 m, was repeated at grid resolutions ranging from 0.25 m to 4 m. In Figure 12 the percent relative error in the surface density as a function of time and grid resolution is plotted. Error is measured relative to the 0.5 m vertical resolution base case. The K profile parameterization shows a much greater sensitivity to resolution than M-Y does. At low-resolution entrainment over the three day simulation can be reduced by as much as 40%. At very high resolution it is enhanced by 15 to 20%. The Mellor-Yamada parameterization, on the other hand, shows generally less than a 5 percent reduction in entrainment at both higher and lower resolutions.

The significant decrease in entrainment at lower resolution with KPP can again be related to the gradient Richardson number mixing parameterization. Because of the finite resolution, shears and vertical gradients in the density field are not necessarily resolved. If the natural system tends to a state where shears and density vary sharply over a 2 m thick region at the base of the boundary layer, a simulation with only 4 m resolution will underestimate both the buoyancy and shear. Since the shear term is squared in the Richardson number calculation the underestimate in the denominator of $Ri_g$ tends to be greater than the underestimate in the numerator. This results in a bias toward overestimating $Ri_g$ as grid resolution decreases.

Increasing the vertical resolution does alleviate this problem, but small grid spacing may also introduce a problem with KPP. The shear mixing scheme responds to low values of $Ri_g$ with instantaneously strong mixing (on the order of 0.005 m$^2$ s$^{-1}$) regardless of the vertical length scale over which the low Richardson number exists. This can result in very strong fluxes of density and momentum just above the pycnocline. At high resolution, this produces intermittent increases in the Richardson number within small portions near the base of the boundary layer which were previously well mixed. This process results in rapid
temporal oscillation in the turbulent mixing coefficients at the boundary layer base as the system alternately overmixes and restratifies there. The fact that the surface boundary layer formulation is matched smoothly with the interior estimate of mixing and that it too depends on a Richardson number based formulation for boundary layer depth exacerbates the problem by extending the oscillations in mixing coefficient throughout the boundary layer. These oscillations are illustrated in Figure 10, which shows time-depth contour plots of the time rate of change of density and the components of the vertical diffusion term in the top 14.5 m of the water column for the $N_e/100$ surface boundary layer simulations.

**Figure 10.** Time-depth contour plots of the time rate of change of density and the components of the vertical diffusion term in the top 14.5 m of the water column for the $N_e/100$ surface boundary layer simulations.

**Figure 11.** Time series of change in surface density with three different initial stratifications for simulations with the two mixing schemes for simulations with a “pulsed” wind forcing.
tions can be decreased by reducing the time step or lowering the maximum shear mixing coefficient \( \nu_{os} \), but neither solution is satisfactory in general. Limiting the time step to the degree necessary to prevent the oscillations can become prohibitively expensive for three-dimensional calculations. Reducing \( \nu_{os} \) weakens the mixing produced by the scheme everywhere that the shear mechanism is active. Thus model resolution must be carefully chosen.

A further complication to these issues for the coastal ocean is the common use of terrain-following coordinate systems. A typical domain can range from 15 cm vertical resolution at the coastal boundary to 10 m resolution offshore. In such a setting, spatial gradients in mixing can develop because of these numerical issues alone. The most promising solution to these issues would seem to lie in finding an alternative to the Richardson number based approach for the interior.

5. Interacting Surface and Bottom Boundary Layers: Case 2

Next the vertical mixing in a one-dimensional water column in which surface and bottom boundary layers are in close proximity is examined. In these experiments both stratification and water depth are varied. The wind forcing leads to the deepening of the surface boundary layers as in case 1, but because of the constraint of zero net horizontal transport, a bottom boundary layer in which the flow is reversed also forms. In all sensitivity studies the response of the bottom boundary layer was observed to be analogous to that of the surface one. That is, while the two boundary layers were not close enough to be affecting each other (greater than approximately 1 m of stratified water between them), the entrainment and deepening of each followed the patterns observed for case 1. The bottom boundary layer produced by M-Y entrained more and extended higher than KPP in strong stratification, and less when the vertical density gradient was low. At the highest initial stratification examined \( (N_o) \), the surface and bottom boundary layers remained separated by several meters of strong stratification for the duration of the simulations for all water depths considered. For weaker vertical density gradients interaction between the boundary layers was observed.

Figure 13 depicts time-depth contour plots for simulations with \( N_o/10 \) stratification in water depths of 15, 17, 18, and 20 m. At 20 m water depth the basic behavior of case 1 can be observed. There is greater deepening and entrainment with the Mellor-Yamada scheme into both the surface and bottom boundary layers. By day six the stratification in the pycnocline has intensified significantly to approximately 6 times the initial value. For shallower water depths, under the same forcing conditions the response gradually changes particularly with the M-Y scheme. The intensified pycnocline that develops between the boundary layers gradually moves upward with the M-Y parameterization but not with KPP. This process reduces the depth of the surface boundary layer while increasing the height of the bottom boundary layer. While the KPP simulations for these water depths show slower growth of the boundary layers and, consequently, less interaction, there is no indication even in the 15 m depth simulation that the pycnocline will move upward when the top and bottom turbulent regions come in close proximity.

The shallowing of the pycnocline with the Mellor-Yamada scheme occurs because the tendency of the bottom boundary layer to grow exceeds that of the surface boundary layer. The gradient in the mixing coefficient at the top of the bottom boundary layer forces the stratification to intensify in the pycnocline to an extent such that at the bottom of the surface boundary layer

\[
\frac{\partial K_r}{\partial z} < K_p \frac{\partial^2 \rho}{\partial z^2}.
\]  

This results in a net upward flux of density. Figure 14 shows profiles of the vertical mixing coefficient for the two schemes for the 15 and 18 m water depth \( N_o/10 \) simulations. In M-Y the strong upward movement of the pycnocline in the 15 m case can be compared to the negligible movement in the 18 m case (Figure 14). For the shallower water column the bottom boundary layer is confined to a thinner layer. Velocity shears at the top of the boundary layer are higher and turbulent kinetic energy production is enhanced. This results in a stronger \( - \frac{\partial K_p}{\partial z} \) at the top of the bottom boundary layer than at the bottom of the surface one. For a water column just 3 m deeper the
result changes. Not only is there more stratification to build the intensified pycnocline, but the bottom boundary can also become thicker (with reduced shears) before impinging on the surface one.

[63] If the bottom boundary layer is not confined to a smaller vertical extent than the surface boundary layer, this response is not observed. Several experiments were tested where the model was initialized with a 7.5 m thick well mixed bottom layer with a pycnocline of several meters thickness between them. In these experiments the pycnocline intensified at middepth and never elevated as the boundary layers that developed were nearly identical.

[66] The KPP response in shallow water is different. The profile of mixing coefficient (Figure 14) for the 15 m depth case shows lower values than the M-Y parameterization throughout the bottom boundary layer for days 3 through 6, while it shows values nearly equal to or greater than M-Y for the 18 m depth case. The profile produced by KPP is only enhanced by increased bottom stress, increased boundary layer depth or an increased gradient in mixing coefficient at the interface with the pycnocline. The sharp onset of mixing with reduction in $R_i_g$ specified by the shear generated mixing function ensures that the vertical mixing coefficient will change rapidly wherever a strong transition in stratification exists without adequate shear. This occurs in the 18 m deep simulation at the interface with the pycnocline of both the surface and bottom boundary layers. The gradient Richardson number changes from greater than $R_i_g$ to close to zero over a single grid point resulting in the same slope at the interior edge of the boundary layer regardless of the particular thickness of the well mixed region. This conclusion holds if the boundary layer depth estimate from the bulk Richardson number calculation does not place the matching point so far into the stratified region that $\frac{\partial K_r}{\partial z}$ is close to zero there. This was not found to occur often in these studies.

[67] The profiles produced with KPP for the 15 m deep simulations show the vertical mixing coefficient diminishes more gradually approaching the pycnocline. In this case the pycnocline has become so thin that it extends over only two grid points. The estimate for the vertical derivative of turbulent viscosity and diffusivity at the boundary layer depth is obtained by interpolating using the value of $K_r$ at the three grid points (vertically staggered with $e$ points) closest to it. For a very thin pycnocline one of these grid points may be in the surface boundary layer while another is in the bottom boundary layer. The net effect is that the estimates of $\frac{\partial K_r}{\partial z}$ for the base of the surface boundary layer can approach zero (because of the gradient at the bottom of the SBL and top of the BBL cancelling out). The estimate at the top of the bottom boundary layer, which uses the updated profile of $K_r$ after the surface boundary layer profile has been determined, will also be reduced from the estimate that would be obtained were the pycnocline thicker or better resolved.

[68] At low enough stratification it would be expected that the enhanced shear that develops at the pycnocline between the surface and bottom layers would be adequate to locally generate mixing across the interface and lead to rapid disintegration of the density gradient. The interior shear mixing formulation of KPP has been utilized

Figure 13. Time-depth contour plots of potential density displaying how the interaction between the boundary layers increases as the water depth decreases. Initial stratification below 7.5 m is set at the $N_r/10$ level.
specifically in the Kantha and Clayson [1994] modifications to the M-Y scheme to enhance mixing in similar situations. Figure 15 displays time-depth contour plots along with profiles at day 4 of the $\rho$ and $u$ component velocity fields for simulations with initial stratification of $N_c/50$. The simulation with KPP develops a well mixed water column approximately a day and a half earlier than the Mellor-Yamada parameterization does. This impacts

Figure 14. Profiles of the viscosity coefficient at days 1–6 for the $N_c/10$ interacting boundary layer simulations.

Figure 15. Time-depth contour plots of potential density anomaly and $u$ velocity component in a 17 m deep water column along with profiles at day 4.
the circulation by shutting down the onshore/offshore
transport.

Figure 16 offers the graphical explanation for this. With surface and bottom boundary layers in close proximity
at day 1.5, adequate shear exists in the pycnocline to
compensate for the enhanced stratification there and reduce
the Richardson number below $Ri_{ig}$. This promotes a
moderately enhanced level of mixing within the pycnocline
and a strong vertical flux of density through the beginning
of day 3. Both the shear and stratification intensify more
with KPP during the first 2 days than with M-Y, but with
KPP the increase in shear is proportionately large enough
relative to the increase in stratification to cause the onset
of turbulent mixing throughout the water column by day 1.2
when $Ri_{ig}$ everywhere in the water column drops below 0.7.
The presence of even moderate mixing across the interface
between the boundary layers with KPP leads to the
"fanning out" of the contours of $Ri_{ig}$ between days 1.2
and 3. With M-Y the shear across the pycnocline never
becomes as intense and mixing proceeds as a boundary
layer entrainment process for significantly longer. Diffusion
of turbulent kinetic energy into this region does not appear
to play a significant role in increasing the mixing either.

The contours of vertical mixing coefficient (Figure 16)
show that the transition from there existing two distinct
boundary layers to a single well mixed water column is more
abrupt with KPP than with M-Y. The maximum $Ri_{ig}$ in the
water column drops from 0.5 to close to 0 within several
hours with KPP while this same process takes over a day
with M-Y. The matching of surface and bottom boundary
layer vertical mixing profiles in KPP likely plays a role in
this as it provides a means by which the diffusion coefficient
at any location feels the effect of mixing elsewhere in the
water column.

6. Two-Dimensional Upwelling: Case 3

A two-dimensional coastal upwelling setting provides
the opportunity to examine how these two mixing schemes
respond when advection also plays an important role in
redistributing density. Sensitivity tests were performed in
which the stratification and the forcing were varied. The
general pattern of response with the two parameterizations
can be characterized by examining a small number of cases.

Figure 17 shows density sections at day five for
upwelling simulations with three different initial stratifica-
tions for the two different mixing parameterizations. In the top two panels the model was initialized with a strongly stratified \((N_o)\) 5 m thick pycnocline between 7.5 and 12.5 m depth, which separates well-mixed surface and bottom layers. The middle two panels are for a case that differs only in that the stratification in the pycnocline is one-tenth as intense \((N_o/10)\). The bottom frames are for a low stratification case \((N_o/100)\) in which the initial stratification is uniform everywhere below 7.5 m depth. All of these were forced with a steady 0.3 dyne wind stress spun-up as described previously. At high stratification the solutions with the two mixing schemes are quite similar. At lower stratification however they differ markedly. In the \((N_o/10)\) case KPP leaves a 5 km region of low-density water trapped near the coast which does not develop with M-Y. The upwelling front with KPP is also approximately 4–5 km farther offshore. In the \((N_o/100)\) case KPP produces a vertically well mixed water column out to 15 km from shore by day 5 while the M-Y scheme allows a shoreward bottom flow that produces stratification to within 5 km of the coast.

Examination of the evolution of the density, across-shore velocity, vertical velocity and vertical mixing coefficient fields over the first four days of the simulations in the \((N_o/10)\) case help to illuminate the relationship between vertical mixing and advection in these systems. Figures 18 and 19 show these fields for simulations with the two parameterizations. These simulations are initialized with a pycnocline that intersects the bottom bathymetry several kilometers offshore. Thus the water column within approximately 2 km of the coast is initially well mixed. In this region, if vertical mixing is sufficiently intense, Ekman transport will be shut down and the Ekman divergence will develop offshore. This response is illustrated with the KPP simulations. A strongly mixed water column develops in the 3–4 km region closest to the coast, shutting down the cross-shore circulation. Intense vertical velocities develop as a result where this Ekman divergence occurs. The pycnocline upwells to the surface at this position and moves offshore. A region of well mixed “surface layer” water is trapped at the coast. A single surface-to-bottom boundary layer extends offshore with the migration of the upwelling front. Intense vertical advection persists at the position offshore where this well-mixed circulation ceases and the two-layer onshore/offshore flow begins.

Figure 17. Potential density anomaly sections for three upwelling simulations with different initial stratification, 5 days after the initiation of the wind stress.
The evolution of the system with the Mellor-Yamada parameterization is different. Although at day 1 the Ekman divergence occurs offshore and a well mixed water column exists in a narrow band at the coast, the shoreward advection of dense water leads to the restratification of the water column in the nearshore environment and the development of a two-layer circulation all the way to the coastal boundary by day 2. Vertical mixing in the nearshore region acts to erode the pycnocline as it upwells, leading to the trapping of some intermediate density water at the coast. Vertical mixing is not strong enough to thoroughly mix the water column. This type of response with the M-Y parameterization has also been noted in a paper by Austin and Lentz [2002]. The region of Ekman divergence that is produced is broader, leading to weaker vertical velocities at the upwelling front.

With KPP, strong vertical mixing in the nearshore acts in conjunction with a large vertical advective transport to thwart shoreward movement of dense water in the bottom boundary layer. This leads to the upwelling front breaching the surface offshore. In contrast, with M-Y, vertical mixing close to the coast becomes inhibited by the bottom shoreward transport of dense water and the front reaches the surface at the coast. So the question that remains is why vertical mixing is not significantly inhibited by the shoreward flow in one case but not the other? The answer rests in the ability of the two schemes to sustain mixing as horizontal advection acts to restratify the water column. Shear production is suppressed by buoyant effects with the M-Y scheme above approximately $Ri_g = 0.21$ (ignoring the effects of diffusion and advection of TKE which are small here). Thus the magnitude of the mixing coefficient will decrease with time in any environment in which the Richardson number is greater than this value. In contrast, the value of the mixing coefficient varies gradually with KPP between $0 \leq Ri_g \leq 0.5$. The mixing intensity is a function of the Richardson number but not its time evolution in KPP, as it is in M-Y. A small change in $Ri_g$ leads to a small change in the magnitude of the vertical mixing coefficient. With M-Y, on the other hand, a small change in $Ri_g$ (particularly between 0.15 and 0.25) can lead to a very large change in $K_r$, as there is a transition from turbulent kinetic energy production to decay. As a consequence of this, vertical mixing can persist and counter the restratifying effect of advection with the KPP scheme in this environment where in M-Y it cannot. The M-Y scheme allows for conditions in which strong mixing develops and stratification erodes ($Ri_g < 0.21$) and ones in which mixing decays and stratification strengthens ($Ri_g > 0.21$) but does not promote the “intermediate” state in which moderate mixing persists with sustained weak stratification. Figure 20 demonstrates this difference. Plotted are cross-shore sections of the gradient Richardson number for the two schemes at days 0.5, 1.0, 2.0 and 3.0 from the coastal boundary out to 15 km offshore. Contours are only plotted for the range $0.0 \leq Ri_g \leq 0.7$. The KPP scheme erodes the pycnocline in a
region characterized by gradient Richardson numbers in the intermediate range $0.2 \leq R_{ig} \leq 0.7$. This is visible in the figure as a region of intermediate $R_{ig}$ approximately 2 km wide that moves offshore with the upwelling front. No similar pattern is present with M-Y because mixing tends to decay rapidly in such an environment rather than promote moderate levels of mixing. Note that a similar response was present in Figure 16 for the one-dimensional interacting boundary layer simulation in which the pycnocline was completely eroded significantly faster with KPP.

A small modification to the interior shear mixing formulation of KPP can make it produce qualitatively similar behavior to M-Y here. The difference in the solutions rests in the rate at which vertical mixing decays in the presence of stratifying effects. Here the KPP shear mixing formulation is modified so as to force the shut down of vertical mixing more abruptly with an increase in the gradient Richardson number. The altered formulation of equation (25) is

$$v_{is} = \begin{cases} v_0 & R_{ig} < 0, \\ v_0 \left[1 - \left(R_{ig}/R_{i0}\right)^6\right]^3 & 0 < R_{ig} < R_{i0}, \\ 0 & R_{ig} > R_{i0} \end{cases}$$

where $v_0 = 5.0 \times 10^{-3}$ m$^2$ s$^{-1}$, $R_{i0} = 0.7$. (Only the exponent on the term $R_{ig}/R_{i0}$ has been changed from 2 to 6 in this modification.) Figure 21 displays the relationship between $K_{is}$ and $R_{ig}$ for the original and altered formulations.

Density anomaly sections are plotted for the KPP formulation with the original shear mixing formulation and with this modified formulation along with results from the M-Y simulation in Figure 22. The early development of the density structure with the modified formulation look similar to the standard formulation. The shoreward edge of the stratification is eroded in both cases, while not with M-Y, because they allow mixing to persist at a higher $R_{ig}$, deeper into the stratified bottom front. By day 1.5, however, the effect of the more abrupt shut down of mixing with the altered scheme is apparent as stratification has intensified where in the standard scheme it has weakened. At days 2 and 2.5 the bottom front is moving shoreward similarly to in the M-Y case. The thicker bottom boundary layer formed with KPP leads to lower bottom velocities and less rapid movement however. At day 2.5 vertical mixing in the nearshore region is eroding the stratification for the modified KPP simulation similarly to what occurred with M-Y slightly earlier. It is important to note that the fact that KPP mixes more strongly at higher gradient Richardson number is not responsible for eroding the stratification before the upwelling reaches the coast. Indeed, the modified KPP formulation actually mixes more strongly at higher Richardson number.
than the original scheme. Rather it is the rate at which mixing diminishes under the influence of restratifying horizontal advection that determines the behavior.

7. Conclusions

This study focuses on finding points of difference between two vertical mixing parameterizations applied to idealized coastal oceanographic settings. The two schemes that are compared were chosen because they represent alternative approaches to the closure problem which have both had some success. The Mellor-Yamada level 2.5 closure has been commonly used in regional continental shelf modeling. The K profile parameterization though tested in numerous open ocean settings, is found to have some shortcomings for application to coastal environments. In particular, the scheme as originally formulated, inaccurately represents mixing in bottom boundary layers and in nearshore regions where surface and bottom boundary effects interact strongly. To ameliorate this situation a bottom boundary layer parameterization modeled after that for the surface boundary layer is appended to the model.

This modified formulation is compared with the M-Y scheme in a series of idealized experiments relevant to the coastal ocean. These include a test of the one-dimensional surface boundary layer response to wind deepening, the evolution of the pycnocline in one-dimension in shallow water when surface and bottom boundary layers are in close proximity, and a two-dimensional study of upwelling on a shallow continental shelf.

In several instances the differences that are observed in the response of the two schemes are related to how each expresses the dependence of vertical mixing on the gradient Richardson number. In the surface boundary layer experiments the formulation of the KPP scheme which allows higher levels of mixing to exist at a higher gradient Richardson number, promotes greater entrainment and more rapid deepening when the stratification of the pycnocline is moderate to low. At low stratification, in the interacting boundary layer experiments the KPP scheme erodes the pycnocline significantly more quickly than M-Y, producing a well-mixed water column much earlier. This again is related to the stronger response to interior shear mixing in the parameterization. The suppression of turbulent production for $Ri_g > 0.21$ has been suggested as a weakness of the M-Y scheme. Several authors have modified it or developed other second-order closure schemes which formulate the relationship between Richardson number and mixing such that turbulence can persist at a higher value [Kantha and Clayson, 1994; Burchard and Baumert, 1995; Mellor, 2001; Canuto et al., 2001]. They have argued that these formulations improve estimates of entrainment when compared to open ocean mixing experiments. It is likely that they also lead to mixing more comparable to KPP in the low-stratification one-dimensional experiments examined here.

![Figure 20. Sections of the gradient Richardson number field for the two schemes at days 0.5, 1, 2, and 3.](image)

![Figure 21. Relationship between gradient Richardson number and vertical viscosity coefficient for the regular KPP scheme and a modified version that shows behavior in the 2-D upwelling simulations more similar to M-Y.](image)
There is no consensus on which representation is the most accurate.

In the interacting boundary layer experiments with higher initial stratification, when the bottom boundary layer is confined to a thinner layer than the surface one, there is a tendency with M-Y for the pycnocline to shallow reducing the depth of the surface boundary layer. In this case the vertical gradient in the mixing coefficient steepens at the top of the bottom boundary layer because of the accumulation of turbulent kinetic energy. The KPP scheme shows no such steepening of mixing profiles or raising of the pycnocline because it holds a "fixed" relationship between the magnitude of the mixing coefficient and the gradient Richardson number. When two turbulent regions in a stratified flow impinge on each other it is not clear how to parameterize the combined effects in the intermediate region in the KPP model. In the enhanced formulation of the KPP model a simple matching across the two layers was assumed. Future research via laboratory studies or direct numerical simulation can be directed at uncovering the characteristics of the turbulence in this complex regime to develop a more physically based formulation.

The two dimensional upwelling simulations also direct attention to differences related to the gradient Richardson number/vertical mixing relationship. At moderate and low stratification, KPP sustains the well mixed nearshore region for the duration of the experiment. This leads to the upwelling front first appearing at the surface some 7 km from the coast and to the trapping of low-density water shoreward of this position. With M-Y, surface-to-bottom vertical mixing is suppressed by the shoreward flow of dense water near the bottom. As a result the two layer flow extends to the coastal wall and the front first breaches the surface there. This difference arises because of the way the two schemes respond to restratifying effects. Whereas with KPP a small change in $R_{ig}$ is directly associated with only a moderate change in the vertical mixing coefficients, with M-Y this same change can mean the difference between sustained turbulent production and decay. As a consequence, the change in $K_r$ over time can be quite large. By maintaining moderate mixing in the nearshore region, KPP effectively “out competes” the restratifying effect of the shoreward bottom flow of dense water under certain conditions, shutting down across-shore transport. While this

Figure 22. Sections of potential density anomaly for the enhanced KPP scheme, for the version with a modified formulation for interior shear mixing, and for the M-Y parameterization at days 1, 1.5, 2, and 2.5.
response is not related to the value of $Ri_p$ at which mixing is suppressed but rather its rate, some of the modified and alternate second-order closure formulations, which change how abruptly turbulence decays under the influence of stratifying effects, may produce solutions in the two-dimensional upwelling setting more akin to KPP.

[82] The one-dimensional wind deepening study reveals that which scheme mixes more is dependent on both the wind stress magnitude and intensity of stratification at the base of the boundary layer. When boundary layer deepening is halted by the development of intense stratification at the pycnocline, entrainment is sustained with the M-Y scheme by downward diffusion of turbulent kinetic energy. No analogous process is represented in KPP. It is unclear in this case which behavior is more appropriate as the self transport of turbulence in such an environment is not yet well understood. While KPP contains no formulation for this process, shortcomings of the M-Y formulation for this term have also been suggested [Stacey et al., 1999].

[83] The two-dimensional coastal upwelling experiments reveal that for moderate to low stratification the choice of parameterization had a significant impact on the resulting circulation close to shore, while at higher stratification, vertical mixing plays a weaker role producing nearly identical solutions with the two schemes. It is important to note that while this study emphasizes cases in which the forcing, initialization and physical constraints lead to significant differences in the performance of the two schemes, in many cases the differences produced may be negligible as their basic response to shear and stratification are qualitatively quite similar.

[84] It would be useful if the differences found between the performance with the two parameterizations suggested one to be superior to the other. Indeed, in some aspects the additional physics included in the M-Y scheme, such as diffusion of TKE or buildup of TKE in a vertically confined boundary layer, seem intuitively more correct. However, a conclusive statement cannot be made without comparisons with real data and/or high-resolution numerical studies that resolve the fine-scale structure of these processes. Hopefully the points of difference noted in this study can help direct such future efforts.

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