Secondary circulation in a region of flow curvature: Relationship with tidal forcing and river discharge

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[1] On the basis of a 301 day acoustic Doppler current profiler mooring in an estuarine tidal strait the strength and structure of secondary circulation in a region of flow curvature is related to variations in tidal forcing and river discharge. During low-flow conditions the structure of secondary flow is consistent with a centrifugally forced helical flow, with bottom flow toward the inside of the bend and surface flow toward the outside of the bend. The strength of secondary flow increases linearly with tidal range and is consistent with a vertical eddy viscosity that is linearly dependent on tidal current speed. During times of high river discharge the strength of secondary flow is significantly reduced, and its vertical structure undergoes a fundamental change over the spring/neap cycle. During spring tides the classic helical flow pattern is evident, albeit weaker than during low-flow conditions. However, during neap tides a more complex two-cell structure is evident. The change between these two states occurs with a spring/neap transition in the subtidal flow, indicating that it is also accompanied by changing stratification. Simple scaling analysis suggests that during weakly stratified conditions, secondary circulation will influence stream-wise dynamics and dispersion for channels with widths on order or less than 0.1 H/Cd, where H is the water column depth and Cd is a quadratic bottom drag coefficient. In contrast, during highly stratified conditions, lateral excursions due to secondary flows are limited to approximately one tenth of the channel’s width and are an ineffective lateral mixing agent.


1. Introduction

[2] The effective along channel dispersion in an estuary is perhaps the single most important parameter to quantify because it, along with the river flow, determines the length of an estuary and is needed to predict the spatial structure of material associated with point and non-point sources. In partially mixed estuaries vertical shear dispersion is often considered to be the dominate process driving horizontal dispersion [Wilson and Okubo, 1978]. From analysis of data collected from a dye injection in the York river estuary, Wilson and Okubo [1978] developed an analytical model to explain the along channel spread of the dye patch. Their model considers both the steady vertical shear associated with the tidal mean velocity as well as the oscillatory vertical shear driven by tidal currents.

[3] However, other processes may also drive along channel dispersion. For example, Fischer [1973] described a shear dispersion mechanism whereby along-channel dispersion is driven by the interaction between lateral mixing by secondary flows of axial gradients of salt and momentum.

This longitudinal dispersion is an elaboration of Taylor’s [1954] model for shear dispersion. Smith [1976] provides a more detailed theoretical analysis of this process in well mixed systems in which lateral mixing rates are determined by the strength of secondary circulation and demonstrates that lateral shear dispersion can be an important contributor to stream-wise dispersion. Recent observations by Trowbridge et al. [1999] suggest that lateral mixing driven by secondary flows may play an important role in defining the along-channel dynamics in partially mixed estuaries.

[4] A number of processes drive secondary circulation in estuarine systems. Nunes Vaz and Simpson [1985] describe secondary flows in well-mixed estuaries as characterized by a pair of surface convergent counter-rotating cells. These axial convergent flows produce streaks of flotsam often observed on flood tide in well-mixed estuaries. The lateral circulation is baroclinically driven by cross-channel salinity gradients set up by stronger tidal currents in the channel’s center acting on the along channel salinity gradient.

[5] The Coriolis effect and flow curvature also drive secondary flows. The forcing term that drives lateral circulation is u(R + f) where u is the along channel flow, F the Coriolis frequency and R the radius of curvature. While in a depth-averaged sense this forcing can be balanced by a
cross-channel sea level slope, vertical shear gives rise to a depth-dependent forcing that drives lateral circulation. The relative effect of curvature to Coriolis can be expressed as the Rossby number $u/R$. At large Rossby numbers curvature dominates rotation and drives a secondary circulation cell flowing toward the inside of the bend at depth and away from the bend at the surface. This flow structure is independent of the direction of the stream-wise flow. For smaller Rossby numbers, Ekman dynamics dominate and the sign of the secondary circulation changes between flood and ebb. For intermediate Rossby numbers, secondary flows are enhanced on the phase of the tide where rotation augments flow curvature. Note that the relative importance of the Earth’s rotation on driving lateral flow is independent of channel width.

[6] The strength and structure of secondary flow is modified by stratification. Geyer [1993] points to two synergistic mechanisms whereby stratification increases secondary flows in large Rossby number environments. First, because vertical density stratification reduces vertical viscosity it supports stronger vertical shear in the stream-wise flow and enhances the forcing to the secondary flows. Secondly, if secondary flows are balanced by a vertical stress divergence then cross-stream shears must increase with decreasing viscosity to generate the vertical stress divergence required to balance the forcing.

[7] On the other hand stratification can reduce secondary flows. A classic example is the arrested Ekman layer whereby cross-slope advection of density in the bottom boundary layer gives rise to buoyancy forces that opposes the cross-slope motion [MacCready and Rhines, 1991, 1993; Garrett et al., 1993]. This is supported in both field and laboratory observations. For example, during highly stratified conditions in the Hudson River estuary [Chant and Wilson, 1997] curvature-induced secondary flows are reduced and even shut down by sloping isopycnals, which are tilted by the secondary flow itself. Johnson and Ohišen [1994] performed laboratory experiments with stratified exchange flow, in which they observed secondary circulation due to Ekman transport between the two layers. In this case a pair of counterrotating cells developed with strong vertical shears across the pycnocline. A similar regime is hypothesized to occur in estuaries by Mertz and Gratien [1995] and Ott and Garrett [1998]. Seim and Gregg [1997] suggest that extreme cross-channel tilting of isopycnals by secondary flows may lead to overturning and vertical mixing.

[8] While secondary flows are an order of magnitude weaker than along channel flows, even weak secondary flows acting on axial gradients in salt and momentum have the potential to significantly impact estuarine stream-wise dynamics and dispersion. Numerous observations report appreciable transverse gradients in velocity and salinity [Nunes Vaz and Simpson, 1985; Valle-Levinson et al., 2000; Wong, 1994; Valle-Levinson and Lwiza, 1995; Geyer and Nepf, 1996]. Often the cross-channel gradient is as large as or larger than the along channel gradient [Chant and Wilson, 1997] and yet the impact of secondary flow on the stream-wise dynamics has not been quantified. Geyer et al. [2000] and Trowbridge et al. [1999] speculate that advective processes associated with secondary flows may play an important role in the momentum balance, but their observations were not detailed enough to test this conjecture.

[9] In this paper the strength, structure and character of secondary flow forced by flow curvature are related to variations in river discharge and tidal forcing. While these observations do not permit an assessment of the impact that secondary flows have on dispersion, results do provide insights into the dynamics of secondary flows and guide a simple scaling analysis that suggests conditions when secondary flows may impact stream-wise processes. The characterization of secondary flows is based on a long term current meter mooring data set collected in the Kill van Kull (Figure 1), a tidal strait connecting Newark Bay to New York Harbor. The mooring was located at the western end of the strait and fluid passing the mooring during ebb, when the flow is to the east from Newark Bay into New York Harbor, has experienced strong flow curvature that drives secondary circulation [Kalkwijk and Booij, 1986; Geyer, 1993]. Tides in this system are strongly semidiurnal with a strong tidal month variability. The tidal range is less than 1 meter during neap tides and approaches 2 meters during spring tide. The Passaic River is the largest source of fresh water to Newark Bay.

2. Data Set

[10] The centerpiece of this data set is a 301 day segment of moored acoustic Doppler current profiler (ADCP) data collected by the National Ocean Service as part of the Physical Oceanography Real Time System (PORTS). This program provides real-time current, water depth and salinity data to shipping traffic in this and other major U.S. ports. Included in the New York Harbor PORTS system was a bottom mounted 1200 kHz RDI ADCP deployed on the western end of the Kill Van Kull in the center of the channel underneath the Bayonne Bridge (Figure 1). The data presented in this paper focuses on a nearly continuous 301 day record collected between November 24, 1997, and September 14, 1998. This segment was chosen because it is the longest nearly-continuous record in the data set. On September 14, 1998, the mooring was lost and has yet to be replaced. While there are a few gaps in the 301 day data set none last more than 2 days, and most are just few hours. These gaps are filled by fitting a mean flow and tidal constituents to 36 hours of data on either side of the gap. The gap is then filled by allowing these constituents to linearly vary with time across the gap, from those obtained prior to the gap to those obtained after the gap. Tidal constituents include diurnal, semi-diurnal, quarter-diurnal, sixth-diurnal and eighth-diurnal motion. Low-pass filtered data is obtained with a Lanczos window with a half power cutoff of 32 hours and a half-window width of 71 hours.

[11] Sea level data is obtained from a NOAA tide gauge at the Battery (Figure 1). Discharge data was obtained from the US Geological Survey from the Passaic River at Little Falls, New Jersey. Over the past century the mean discharge of the Passaic River, recorded at Little Fall New Jersey is 60 m$^3$/s. During this same period flow exceeded 90 m$^3$/s 20% of the time and was less than 25 m$^3$/s 20% of the time. Discharge during the PORTS ADCP deployment was characterized by approximately 100 days of high
river discharge and 200 days of low river discharge (Figure 2).

3. Subtidal Flow Variability

[12] While analysis in this paper focuses on tidal period motion, and in particular in the flow structure during ebb when fluid passing the mooring has experienced substantial curvature and secondary flows are evident, I begin with a characterization of the low-passed currents for they are useful in characterizing the spring/neap variability. Later, the observed spring/neap variability in these tidally mean flow is used as a proxy for stratification. This proxy is used to interpret spring/neap variability in the strength and structure of the secondary flow.

[13] The shear in the estuarine flow is estimated by the difference between the low-passed current at a bin 13 meters above the bottom (mab) and a bin 4 mab (Figure 3). In this reach estuarine circulation is characterized by a mean surface layer flow to the east, out of Newark Bay, and a bottom...
flow to the west into Newark Bay. The bin 13 mab represents the upper most reliable ADCP bin, while the bin 4 mab typically had the strongest westward mean flows. High shears characterize times of stronger two-layer estuarine circulation, and weak shears represent times of weak estuarine circulation. Time series of the depth averaged low-passed shear along with the tidal range shows that the residual shear has a high frequency component and a lower-frequency variability that is associated the spring/neap cycle (Figure 3). The spring/neap variability is particularly evident during the stronger spring tides on days 425, 458, 481, and 515 when residual shear goes to zero. In contrast, stronger shear occurs during neap tides, and weak spring tides, such as around days 470, 500, and 525. A clearer relationship between residual shear and tidal range is evident by binning the data as a function of tidal range (Figure 4). The data was further binned into two groups as a function of river discharge. Times of high discharge corresponded to flow at Little Falls that are greater than 40 m$^3$/s, while low discharge is defined as flows that are less than 40 m$^3$/s. From the 301 day ACDP record approximately 200 days were into the high discharge category and 100 in the low discharge category. Residual shears are stronger during high-flow neap tide conditions when mean shear exceeds 10 cm/s across the water column. This is approximately twice the shear that occurs during low-flow neap tide conditions. During both the wet and dry period mean shear is appreciably reduced during spring tides, and even reverses during the dry period. This reversal is likely due to a mean inflow that is sheared by a bottom stress that penetrates the entire water column during spring-tide low-flow conditions.

The relationship between low-frequency shear and tidal range is even more concisely described by defining the depth averaged mean shear as $U_\tau = \sqrt{\left(U - U_0\right)^2}$ where $U$ and $U_0$ are the low-passed depth-dependent and depth-averaged along-channel flow respectively, and the integral is taken over ADCP bins 1–12. At the mooring location there is a mean Eulerian inflow of $\sim 3$ cm/s that is relatively independent of tidal range or river discharge (Figures 5a and 5c). However, a clear dependence of the low-pass shear, $U_\tau$, on tidal forcing is apparent during times of high and low river discharge (Figures 5b and 5d). The smooth relationship between estuarine shear and tidal forcing suggests that the “noise” associated with the higher frequency component of the mean shear (Figure 3) has been significantly reduced by the binning process due to the length of the ACDP record.

During times of high river discharge the spring/neap transition occurs at a tidal range of approximately 1.5 meters while for the low discharge case it occurs at a slightly lower tidal forcing. Numerous other studies indicate that spring/neap transitions are associated with a marked change in stratification [Geyer et al., 2000; Peters, 1997; Hass, 1977]. Since salinity stratification data is not available for this study, spring/neap variations in mean shear is used as a proxy for the spring/neap changes in stratification. Later, this proxy is used to interpret the changing character of the secondary flow.

### 4. Secondary Flows During Ebb

Current vectors during maximum ebb as a function of tidal range for the wet period and dry period are shown in Figure 6. During the wet period the flow exhibits little veering with depth. In contrast, appreciable veering is evident during the dry period. The veering is consistent
Figure 4. Low-passed flow binned as a function of tidal range during (top) high-flow conditions and (bottom) low-flow conditions. Contours depict current speeds in m/s.

Figure 5. (a) Lowpassed depth-averaged flow binned as a function of tidal range during high-flow conditions. (b) Shear in low-passed currents as function of tidal range for high-flow conditions. (c) Low-passed depth-averaged flow binned as a function of tidal range during low-flow conditions. (d) Shear in low-passed currents as function of tidal range for low-flow conditions.
with the classic helical flow associated with a curving, vertically sheared flow [Kalkwijk and Booij, 1986; Geyer, 1993], with the surface fluid flowing toward the outside of the bend and the bottom currents toward the inside of the bend.

Details of the flow structure on ebb are more evident in contours of along-channel and cross-channel current speed as a function of tidal range and depth during low-flow (Figure 7) and high-flow (Figure 8) conditions. Here the along channel direction is defined as the direction of the depth-averaged flow during maximum ebb. During both periods vertical shear in the along channel flow is intensified near the bottom. Vertical shears are slightly stronger during the wet period, resulting in stronger surface flows on ebb. The lateral flow during the dry period is consistent with Figure 6a, both showing evidence of helical circulation during neap and spring tides. During spring tides the strength of the lateral circulation exceeds 10 cm/s with bottom flows to the north, toward the inside of the bend, and surface flows to the south, toward the outside of the bend.

Figure 6. Currents during maximum ebb binned as a function of tidal range during (top) wet period and (bottom) dry period. Surface currents tend to be stronger (longer sticks), while bottom currents are weaker (shorter sticks). Note that rotation of current with depth changes over spring-neap cycle during wet period.

Figure 7. (top) Along-channel flow and (bottom) cross-channel flow during maximum ebb as a function of tidal range for low flow conditions. Contours show current speed in m/s. Positive cross-channel flows are to the north (toward the inside of the bend). Thick line in lower panel depicts isotach where cross-channel flows vanish.
Secondary flows are weaker during the wet period (Figure 8), when cross-channel flows are generally less than 4 cm/s. The vertical structure of the secondary flows is more complicated during the wet period, relative to the dry period, yet it is coherent and its character undergoes a fundamental change at a tidal range of 1.5 meters. Note that this is the tidal range that the mean along-channel flow undergoes a spring/neap transition. When tidal range exceeds 1.5 meters, the classic helical single-cell secondary flow structure is evident, with a southward component of surface flow toward the outside of the bend and a northward component to the bottom flow toward inside of the bend. However, when the tidal range is less than 1.4 meters, the vertical structure of the lateral circulation is suggestive of a two-cell lateral flow, with bottom and surface flows toward the outside of the bend and flow in the interior toward the inside of the bend. Furthermore, during high river discharge conditions there is little change in the intensity of secondary flows while the tidal range is less than 1.5 meters. This is in contrast with the dry period, which shows a gradual increase in the strength of secondary flows with tidal range.

The relationship between tidal forcing and secondary circulation is characterized more quantitatively by defining the intensity of vertical shear during maximum ebb in the along-channel flow as $u_z = \frac{\sqrt{\int (u-a)^2 \, dz}}{C_0}$ and vertical shear in the cross-channel flow as $v_z = \sqrt{v^2 + \partial_z u^2}$, where $u$ and $u$ are the depth dependent and depth averaged along-channel flow and $v$ is the cross-channel depth-dependent flow. Since the along-channel direction is taken along the direction of the depth-averaged flow, by definition the depth average of $v$ is zero. The strength of secondary circulation is therefore concisely represented by $v_z$.

Results of this analysis applied to the ADCP data collected during the wet period are shown in Figure 10. Similar to the dry period, depth averaged flows during the ebb are linearly related to tidal range, and vertical shear in the along-channel flow is also linearly related to tidal current speed, although the shear is stronger than during the dry period. It is likely that the increase in vertical shear is supported by increased stratification during times of high river discharge.

However, the relationship between tidal forcing and secondary flow is fundamentally different during the high-flow condition. The relationship between vertical shear in along-channel flow and that of the cross-channel flows is more complex, with the vertical shear in the along-channel flow increasing linearly with tidal range, while the vertical shear in the cross-channel flow increases nonlinearly. This suggests that the vertical shear in the along-channel flow is more strongly influenced by the tidal forcing, while the vertical shear in the cross-channel flow is influenced by a combination of tidal forcing and river discharge.

The relationship between tidal range, tidal currents, vertical shear in the along-channel currents and cross-channel flows during the dry period on maximum ebb is shown in Figure 9. The linear relationship between tidal range and tidal current speed (Figure 9a) is to be expected. For neap tide conditions, maximum ebb depth averaged velocities are less than 50 cm/s, while during spring tides, when the tidal range at the Battery is 1.8 meters, depth averaged ebb currents exceed 80 cm/s. Vertical shear in the along-channel flow on ebb, $u_z$, also increases linearly with tidal range (Figure 9b). The vertical shear on ebb, $u_z$, is 0.035 m/s during neap tides and increases to 0.055 m/s during spring tides. A linear dependence of the strength of secondary flows, $v_z$, on the tidal current speed is evident in Figure 8c as is the relationship between secondary flow and along-channel shear (Figure 8d). In summary, Figure 8 emphasizes that vertical shear in both the stream-wise flow and the strength of secondary flow increases linearly with tidal forcing.
Figure 9. (a) Along-channel depth-averaged flow binned as a function of tidal range during dry period. (b) Shear in along-channel flow binned as function of tidal current speed. (c) Cross-channel shear as function of tidal current speed. (d) Cross-channel shear binned as function of along-channel shear.

Figure 10. (a) Along-channel depth-averaged flow binned as a function of tidal range during wet period. (b) Shear in along-channel flow binned as function of tidal current speed. (c) Cross-channel shear as function of tidal current speed. (d) Cross-channel shear binned as function of along-channel shear.

(secondary) flow is poor. One robust result, however, is that secondary flows are appreciably weaker during the high discharge period when vertical shear in the lateral flow is 2–3 cm/s, in contrast to the dry period when this shear ranges between 3 and 9 cm/s. The decrease in secondary circulation during the wet period occurs despite the increase forcing due to the increased shear in the stream-wise flow. While the regression suggests that the strength of secondary flow decreases with tidal forcing during times of high river discharge this result should be
forcing. For the linear increase in eddy viscosity, \( Av \), that is linearly proportional with tidal secondary flows, \( (\partial v/\partial z) \), increase linearly with tidal forcing. A quadratic increase in the interior stresses, \( \tau_{bw} \), consistent with the well known quadratic bottom drag formulation. A quadratic increase in bottom stress with tidal forcing is required both for equation (1) to hold requires both friction terms to increase quadratically with tidal forcing. The quadratic increase in bottom stress with tidal forcing is consistent with the well known quadratic bottom drag formulation. A quadratic increase in the interior stresses, \( (\partial \tau/\partial z) \), is also consistent with the observation that secondary flows, \( (\partial v/\partial z) \), increase linearly with tidal forcing. For the linear increase in \( (\partial v/\partial z) \), coupled with an eddy viscosity, \( \Lambda_v \), that is linearly proportional with tidal forcing results in interior cross-stream stresses, \( A_v \), \( (\partial v/\partial z) \), that increase quadratically with tidal forcing. A linear increase in vertical eddy viscosity with tidal range in well mixed conditions is consistent with \( \Lambda_v \) scaling with the product of the water column depth and the tidal current speed. A linear dependence of vertical mixing on tidal current speed in well mixed environments has been deduced from field data in many studies, including the classic work done by Bowden and Fairbairn [1952a, 1952b].

During times of high river discharge the structure of secondary flow is more complex. Despite enhanced vertical shear in the stream-wise flow, relative to low-flow conditions, secondary flows are appreciably weaker during times of high river discharge. This suggests a fundamental change in the momentum balance that governs secondary flows during times of high river discharge, because despite the stronger shears and weaker mixing, secondary flows are smaller. For if equation (1) governed during times of high river discharge, both the increase in vertical shear and decrease in vertical mixing would produce stronger secondary circulation [Geyer, 1993]. In contrast, observations presented here clearly show that secondary flows are reduced during times of high river discharge. This suggests that secondary circulation is being shut down by stratification. While salinity data is not available here such a momentum balance has been shown to shut down secondary circulation in other estuarine systems [Chant and Wilson, 1997].

A second notable difference in the character of secondary flows between the wet and dry period is its vertical structure. During the wet period the vertical structure of secondary flows changes with the spring/neap cycle. During spring tides the flow structure is consistent with the expected helical structure driven by flow curvature, albeit weaker than the lateral circulation during low-flow conditions. In contrast, during neap tide the secondary flow’s structure is suggestive of a two-cell circulation. While such a structure can be driven by an Ekman circulation in exchange flows, as observed in the laboratory experiments by Johnson and Ohslen [1994] and suggested by field observations by Ott and Garrett [1998], the apparent two-cell structure here is likely due to another process, because stream-wise flows are unidirectional.

Rather, I speculate that the complicated cross-channel flows are due to a lateral sloshing associated an internal seiche, such as described by Chant and Wilson [1997]. The seiche is initially set up by the secondary flows which tilts the halocline upwards toward the inside of the bend. The cross-channel baroclinic pressure gradient associated with the tilted halocline tends to shutdown the curvature induced secondary flows. However, while the baroclinic pressure gradient may, at some point, balance the forcing by the curving-sheared flow, secondary flow only begin to decelerate at this point. The result is that the baroclinic pressure gradient overshoots its equilibrium value and set up a wave-like oscillation. As the fluid is advected away from the region of flow curvature the lateral gradient relaxes and possibly rebounds in the form of a lateral internal seiche. One consequence of this lateral sloshing is that, in contrast to the classic secondary flow in the low-flow condition, fluid trajectories are not helical. Rather, fluid parcels laterally oscillate as they are advected down stream. Since the mooring location lies close to the region of flow curvature it is likely that during maximum ebb the mooring remains in the set-up portion of the seiche and consequently a mean, albeit weak, lateral motion is observed.

During spring tide conditions the classic single-cell secondary flow structure is evident, albeit weaker than secondary flows during spring tide low river discharge conditions. The change in the structure of secondary flow, from the two-cell structure to the classic helical flow structure occurs at a tidal range of 1.5 meters, coinciding with the spring/neap transition in mean shear. Both of these changes are consistent with a reduction in stratification, suggestive that during spring tide conditions the water column lacks the buoyancy to shut down secondary flows. This spring/neap change in the strength and structure of secondary flows during high river discharge is consistent with numerous observations showing dramatic fortnightly and tidal monthly changes in stratification in partially mixed estuaries [Hass, 1977; Peters, 1997; Geyer et al., 2000]. However, the observed reduction of secondary flows during spring-tide high-river flow condition relative to low-flow conditions, suggests that lateral baroclinic forces still plays a role in reducing secondary flows.
The change in the character of secondary flows over
the spring/neap cycle may impact stream-wise dispersion
for it would effect lateral mixing processes. In the unstratified
case particle trajectories follow a helical path down
stream of regions of flow curvature. In the absence of
additional channel bends this pattern extends downstream
until friction spins down the secondary flows. The spin
down time of secondary flows is $H/c_D U$ [Geyer, 1993].
Assuming current speeds of 0.75 m/s, a bottom drag
coefficient of $2.5 \times 10^{-3}$ and a depth of 15 meters, the
spin down time is 2.2 hours, corresponding to approxi-
mately 6 km; nearly the length of the Kill van Kull.
Secondary flows that are 10% of the stream-wise velocities
would result in lateral mixing over a length scale of 600
meters. This could be significant in estuarine channels with
widths of 1 km or less, such as the Kill van Kull, Arthur Kill
or the Hudson River (Figure 1). More generally, the width
of secondary flows will effectively mix is of order $\alpha H/c_D$,
where $\alpha$ is the ratio of cross-channel flows to along channel
flows. Kalkwojk and Booij [1986] suggest that $\alpha = 6H/R$. Large
Rossby number environments occur in estuarine
reach where $R$ is of order 1 km, thus for channels with
depths of order 10 m $\alpha$ is approximately 0.06. Geyer [1993]
demonstrates that weak stratification will increase $\alpha$ by a
factor of 2–3. Thus in weakly stratified estuaries $\alpha$ tends to
be around 0.1, and this is in agreement with the observa-
tions presented here during times of low river discharge
(Figure 9). In summary, longitudinal dispersion and stream-
wise dynamics are likely to be influenced by flow curvature
in weakly stratified channels that are of the order of or
narrower than 0.1H/c_D, while channels that are appreciably
wider than 0.1H/c_D are less impacted by flow curvature.

In contrast, in the stratified case where curvature
induced secondary flows are associated with an internal
lateral seiche, particle trajectories are not helical but rather
oscillate laterally while advected down stream. A first-mode
internal lateral seiche would oscillate at period,

\[ T = 2 \sqrt{\frac{H}{\bar{g} H}} \]

where $W$ and $g'\bar{g}$ are the channel’s width and reduced gravity
respectively. Neglecting effects of along channel self prop-
agation, the wave-like seiche would have an along channel
length scale of $L = uT = 2W \sqrt{\frac{H}{\bar{g} H}} = 2WF$ where $F$ is a
Froude number. Thus the cross-channel excursion of fluid
parcels would be ofW. The Froude number during stratified
trends tend be of order 1, suggesting that the lateral
seiching motion would involve lateral stirring only over $\alpha$
of the channel’s width. Observations presented here show
that $\alpha$ tends to be less than 0.05 thus the lateral seiching is
an ineffective lateral mixing agent. This is consistent with
observations by Trowbridge et al. [1999] that suggest that
stream-wise dynamics in the Hudson River estuary are not
modified by secondary circulation during times of strong
stratification (neap tide) yet may impact stream-wise
dynamics during weakly stratified (spring tide) conditions.

6. Conclusions

Observations presented here demonstrate the chang-
ing strength and vertical structure of curvature induced
secondary flows as a function of tidal range and river
discharge. During times of low river flow the structure of
secondary flow exhibits the classic curvature induced hel-
cical flow pattern, with bottom flows toward the inside of the
bend and surface flows toward the outside of the bend. The
strength of the secondary flow increases linearly with tidal
forcing, and is approximately 10% of the along channel
current speed. The strength of both stream-wise and second-
ary shear increase linearly with tidal forcing. Since
secondary flows are forced by the square of the stream-
wise shear, this suggests a quadratic increase in forcing to
secondary flows that is balanced by a quadratic increase in
stresses with tidal forcing. A quadratic dependence of stress
on tidal forcing is consistent with an eddy viscosity that
increases linearly with tidal current speed.

During times of high river discharge the strength of
secondary flows are appreciably weaker than those during
dry conditions, despite an increase in vertical shear. This
suggests that secondary flows are reduced by stratification.
During spring tide conditions secondary flows exhibit a
helical flow structure, with bottom flow toward the inside
of the bend and surface flows toward the outside of the
bend. The flow structure, however, is more complex
during neap tide conditions and while it is suggestive of
a two-cell circulation. It is very weak and probably
changes with distance along the channel due to the setup
and excitation of an internal lateral seiche. The transition
between the single-cell helical flow pattern and the more
complex two cell pattern occurs with the spring/neap
transition in residual flow, suggesting that this change is
associated with changing stratification. During neap tides
stronger stratification provides sufficient buoyancy to shut
down secondary flows while the weaker stratification
during spring tides cannot.

Finally, simple scaling analysis suggests that during
weakly stratified conditions the stream-wise dynamics of
channels with widths on the order of 0.1 C_D/H or less are
impacted by effects of flow curvature, while the stream-wise
dynamics of channels wider than that are less impacted by
flow curvature. In contrast, during highly stratified condi-
tions effects of flow curvature are less likely to impact the
stream-wise dynamics, regardless of channel width, because
the lateral excursions are limited to less than one tenth of the
channel’s width.

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