Lectures 1 and 2: Satellite orbits and Measurement Geometry

Satellite Remote Sensing Systems

The flow of information from land or sea surface to satellite to user depends on many features of the land/ocean phenomena, the observing system, and the intervening atmosphere.

1. Ocean phenomena:
   - color, temperature, roughness, albedo, emissivity, terrain, vegetation canopy, wave height/sea level

2. Water leaving signal:
   - WLR (water leaving radiance) and land surface reflectivity can depend on relative position of Sun and satellite, therefore time-of-day and field-of-view affect signal

3. Satellite sensor:
   - the observation may be from a passive or active system

(4) Resolution: FOV, aperture, scan geometry can depend on satellite trajectory and altitude (orbit), pointing

(5) Geolocation: position, time, pointing. Orbit affects repeat sample interval

(3) Sensor: the data is a measurement or image

(2) Water leaving signal: WLR can depend on relative position of sun and satellite, time of day, emissivity, reflectance

(1) Ocean phenomena: color, temperature, roughness, height

(6) Position of the satellite, or range to target may be the actual data.
• measurement may be an image or point-wise data
• scan geometry depends on satellite trajectory
• remote sensing instruments are not in physical contact with the phenomena under investigation – properties are inferred from the received radiation

4. field of view and ground-track sampling frequency depend on orbital period, trajectory, and altitude

5. orbit determination and satellite pointing information determine the geographic location of observation (time, place, averaging period). (The range to data may be the observation of interest.)

**Satellite position**

Critical to several aspects of the data set acquired by the sensor:

- Sub-satellite ground-track and pointing: data coordinates
- Altitude and field of view (FOV) resolution
  errors, atmospheric interference
- sun-satellite angle illumination of land/sea surface
- satellite velocity and sensor image forming and scan pattern signal to noise
- repeat period repeat sampling interval for time-varying phenomena

**Operational issues**

Higher altitude
  – more energy required from launch vehicle
  – less drag, more stable orbit

On orbit, and getting to orbit – don’t smack into anything else

Sun-satellite angle affects thermal state, available power, solar wind perturbations to orbit stability and electronics, ground communication

Satellite position is determined principally by orbital physics, with influence from satellite dynamics (drag, roll, pitch, moment of inertia). The set of parameters that define an orbit are referred to as the satellite’s *ephemeris*. 
Physics of satellite orbits

50 years before Isaac Newton, Johannes Kepler analyzed data on planetary movements and deduced that:

1. Planets move in elliptical orbits with the sun as one focus
2. the radius vector from the sun to the planet sweeps out equals areas in equal times
3. $T^2 : R^3$ ratio is constant for all planets, where $T$ is orbital period and $R$ is semi-major axis of the orbit

Substitute satellite for planet and earth for sun in the above rules and they apply for artificial earth satellites.

For planets, a convenient unit of time is Earth Years, and for distance a convenient unit is the Astronomical Unit (A.U.) being the distance from the Sun to Earth.

Then, trivially, $R^3 = T^2$ (because both units = 1)

For Mars, the orbital period is 1.88 Earth years, so

$$R = T^{2/3} = (1.88)^{2/3} = 1.52 \text{ A.U.}$$

which is indeed the average radius of the Martian orbit. (see e.g. http://www.windows.ucar.edu/tour/link=/our_solar_system/planets_table.html)

Newton discovered the laws of gravitation and explained planetary and satellite orbits in terms of the balance of forces:

Gravity : centrifugal acceleration

Newton:

$$F = ma = m \frac{dv}{dt}$$

Gravity:

$$F_{gravity} = \frac{GMm}{r^2}$$

where $r$ is the separation distance of masses $M$ and $m$

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

$M_{\text{earth}} = 5.976 \times 10^{24} \text{ kg}$
An object of mass $m$ falling in the Earth’s gravitational field at the Earth’s surface accelerates at a rate determined by

$$ma = \frac{GMm}{r_{\text{earth}}^2}$$

$$a = g = \frac{GM_{\text{earth}}}{r_{\text{earth}}^2}$$

$r_{\text{earth}} = 6373 \text{ km}$

Note: this is independent of mass $m$ - remember Galileo $\Rightarrow$

$$g = 9.81 \text{ m s}^{-2}$$

A satellite in permanent Keplerian orbit maintains a balance between gravity and centripetal force due to its circular motion.

Centripetal acceleration \[\frac{v^2}{r}\]

$v$ is the speed of the satellite

$r$ the radius of the orbit
Acceleration is the rate of change of velocity \( a = \frac{\Delta v}{\Delta t} \)

\[
v = \begin{bmatrix} -\sin \theta, \cos \theta \end{bmatrix}
\]

\[
\frac{dv}{dt} = \begin{bmatrix} -\cos \theta, -\sin \theta \end{bmatrix} \frac{d\theta}{dt}
\]

(notice that vectors \( v \) and \( \frac{dv}{dt} \) are perpendicular)

and \( \frac{d\theta}{dt} = \frac{v}{r} \)

so it follows that the acceleration is

\[
\frac{dv}{dt} = \frac{v^2}{r} \begin{bmatrix} -\cos \theta, -\sin \theta \end{bmatrix}
\]

\[
a = \frac{v^2}{r}
\]

and the acceleration is always perpendicular to the direction of \( v \).

To have a balance between gravitational force and centripetal force in a circular orbit means that:

\[
\frac{GMm}{r^2} = \frac{mv^2}{r}
\]

Note that \( v \) does not depend on \( m \)

\[
v = \sqrt{\frac{GM}{r}}
\]

The satellite velocity is set by the radius of the orbit, which is the radius of the earth plus the satellite altitude above ground.

Altitude of the satellite (altitude = \( r - r_{earth} \)), mass of Earth, and G, determine the satellite’s orbital period:
\[ vT = 2\pi r \]
\[ T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} \]
\[ T^2 = \frac{4\pi^2 r^3}{GM} \]
\[ T = 2\pi \sqrt{\frac{r^3}{GM}} \]

The ISS at altitude 360 km \textit{above the earth surface} (the distance from D.C. to NYC) will have a period of

\[ T = 2\pi \sqrt{\frac{(6373 + 360)^3 \times 10^9}{3.986 \times 10^{14}}} \]

\(<<\) add radius of Earth 6373 km to get radius of orbit

\[ = 5.498 \times 10^3 \text{ s} \]

\[ = 91.6 \text{ minutes} \quad \text{at a speed of } 7.69 \times 10^3 \text{ m s}^{-1} = 17,200 \text{ m.p.h.} \]

D.C. to NYC is 360 km, which at the speed of the ISS takes 46 seconds.

The characteristics of how an earth observing satellite samples the land/ocean surface and atmosphere depends on its orbit with respect to the Earth’s center of mass, and the rotation of the surface of the planet beneath this orbit (the ground track pattern, and repeat period).

Often it is the pattern of the ground-track traced by the satellite that is a key factor in deciding a suitable orbit.

However, the rotation of the Earth \textbf{has nothing to do with the orbit} of the satellite itself, which must be considered in a non-accelerating Newtonian coordinate system fixed w.r.t. the stars.

The three principal orbits employed for Earth observation are:

- Geosynchronous
- Sun-synchronous (polar orbiting)
- Near equatorial low inclination
Geometry of elliptical orbits

Figures from Martin (chapter 1), Kidder and Vonder Haar (their figs 2.4, 2.5) and Stewart (his figs 15.1, 15.3)

Strictly speaking, a satellite orbit is elliptical (and described by 6 Keplerian elements), though in practice most earth observing satellites are in near circular orbits (described by 4 elements).

Orbit shape is defined by its eccentricity \( e \), semi-major axis \( a \)
The location of the orbital plane is defined with respect to an inertial coordinate system – i.e., one that is independent of the Earth’s rotation.

Satellite position is defined in polar coordinates: \[ r, \theta \quad r = \frac{a(1-e^2)}{1+e \cos \theta} \]

where angle \( \theta \) is called the true anomaly.

The true anomaly is sometimes measured counterclockwise from perigee for elliptical orbits, and always measured counterclockwise from the Ascending Node for circular orbits.

**Right-ascension-declination coordinate system**

**z-axis** parallel to Earth rotation axis (north pole – south pole)

**x-axis** toward point on the celestial sphere when the sun is at the vernal equinox (March 21). So x-axis lies in both the ecliptic and the earth’s equatorial plane

![Figure 15.1](image)

Satellite orbit is described by 3 angles:

1. \( i \) = inclination = angle between satellite orbital plane Earth’s equatorial plane
   
   \( i < 90 \) prograde; \( i > 90 \) retrograde

   It follows that \( i \) is also the maximum latitude of the satellite ground track
(2) $\Omega =$ right ascension = angle from x-axis to ascending node $N$, where $N$ is the location on the equator where the satellite crosses from south to north (on the ascending pass).

So $i$ and $\Omega$ specify the position of the orbit plane with respect to the fixed stars.

(3) $\omega =$ perigee angle = angle in orbital plane between $N$ and perigee (for elliptical orbits).

The satellite position $\theta$ is measured counterclockwise from ascending node $N$. (For elliptical orbits it is often measured counterclockwise from perigee.)

In a Keplerian orbit the orbital plane does not rotate. It remains fixed with respect to the distant stars (the inertial reference frame).

If the earth were not rotating the ground track would be a great circle. But the earth does rotate, so the ground track progresses steadily westward as the earth spins a fraction of its full rotation during each repeat of a Low Earth Orbit satellite orbit. (More on this later when we discuss sun-synchronous orbits.)

(prograde orbit)

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Figure 1.2: Mercator map of the satellite ground track for the orbit in Figure 1.1 and for (a) non-rotating Earth; (b) rotating Earth. See text for further description (Adapted from Elachi, 1987, Figure B-6).
Launching into orbit

Launches are usually designed to place the satellite at perigee at the time of main engine cutoff. This requires the least energy.

The launch sequence for a low earth orbit satellite is typically:

- main engine cutoff at around 5 minutes
- second stage at around 10 minutes
- payload fairing is jettisoned after second stage
- separation of satellite from the launch vehicle occurs after about 1 hour
- solar panel deployment, sensor deployment, activation of navigation, checks on pointing and nominal performance etc would proceed over the next 24 hours (or possibly longer)
- NOAA meteorological satellites weigh about 1500 kg, are about 5 m long x 2 m diameter, deploy a solar array 6 m long, and require 800 W power

Launch costs are roughly $5000 per kilogram of payload for low earth orbit, and $20,000 per kg payload for geostationary.

[Links to launch movies in the Powerpoint presentation.]
Satellite orbits and Measurement Geometry, continued...

If the earth were not rotating the ground track would be a great circle.

But the earth does rotate, so the ground track progresses steadily westward as the earth spins a fraction of its full rotation during each repeat of a Low Earth Orbit.

For a prograde orbit:

Ground track and repeat intervals

The separation of equator crossings can be calculated from the period of the satellite and the speed of a point on the equator (circumference of the Earth divided by 1 sidereal day). If the earth circumference divided by the equator crossing separation is an integer, the satellite is in an exact repeat orbit.

There are 365.25 solar days in a year but 366.25 sidereal days

Sidereal day = 365.25/366.25*24 = 23.93 hours

A point on the equator moves at speed:

\[ v_{\text{equator}} = \frac{2\pi R_{\text{earth}}}{(23.93 \times 3600)} = 464 \text{ m s}^{-1} \]

The TOPEX and Jason satellites have a period of about 112 minutes. In this time, a point on the equator moves:

\[ v_{\text{equator}} \times (112 \times 60) = 3122 \text{ km} \]

which is the distance between successive equator crossings. These fill in to complete coverage of the earth with an exact repeat of 9.9156 days.
**Orbit precession**

In a Keplerian orbit the orbital plane remains fixed with respect to the distant stars (the inertial reference frame), but this can be inconvenient for many earth observing systems because it means the local hour-of-day of the observation time varies. This presents some difficulties for constructing the long time series sampled at regular intervals that we typically use in geophysical data analysis.

However, the gravitational potential of the earth is not uniform because the distribution of mass in the earth is not exactly spherical and this leads to slow changes in the position of the orbital plane form the idealized Keplerian case.

The gravitational potential is approximated as:

\[
U \approx -\frac{GM}{r} \left[ 1 - \frac{r_{\text{earth}}^2}{r^2} \frac{J_2}{2} \left( 3 \sin^2 \phi - 1 \right) \right].
\]
where \( r \) is the orbit radius (or semi-major axis), \( r_{\text{earth}} \) is the equatorial radius of the earth, and \( J_2 \) is quadrupole gravitational coefficient (or zonal harmonic coefficient, and is related to spherical harmonics of the gravity field) that contains most of the gravitational anomaly due to the equatorial bulge of the earth (\( J_2 = 1.082 \times 10^{-3} \)).

This irregularity causes a steady precession of the orbital plane about the \( z \)-axis at a rate:

\[
\dot{\Omega} = -\frac{3}{2} J_2 \left( \frac{r_{\text{earth}}}{r(1-e^2)} \right)^2 \cos i
\]

which depends on the satellite orbits inclination, altitude and eccentricity.

**Sun synchronous orbits**

The orbital parameters can be chosen so that the rate of precession of the ascending node is 360/365.25 degrees of longitude per 24 hours, in which case the orbital plane maintains a fixed angle with the line from the sun to the earth throughout the year.

Then the satellite passes overhead at the same local time every day.

The sun synchronous orbit with zero eccentricity has a semi-major axis of 7228 km and inclination of 98.8°.

This is the orbit used by the NOAA “polar orbiting” meteorological satellites, NASA’s Terra and Aqua satellites, the DMSP satellites, Envisat, and many more.

It is conventional to refer to satellites that ascend past the equatorial nodal point between 0600 and 1200 local time as \textit{morning satellites or daytime passes}, and those that ascend between 1200 and 1800 local as \textit{afternoon or evening satellites}.

The NOAA satellites typically pass at around 7:30 am and 1:30 pm local time.

This orbit does not go directly over the poles, so the satellite will not observe the polar region unless the field of view of the instruments on board is wide enough to see to the higher latitudes.

**Geostationary orbits**

Another way to observe the same point on the earth’s surface at the same time every day is to simply park a satellite directly overhead.

A satellite in the equatorial plane (zero inclination), zero eccentricity, and semi-major axis of 42,168 km has a period of 24 hours, and it said to be geostationary.

View satellites and their orbits at:
Shows ground tracks and/or 3-D orbits for most satellites

Notice the clusters of satellites in the major categories or orbits

- Geostationary
- Polar orbiting (NOAA, Topex, ERS, Envisat)
- Low Earth orbit (Iridium, HST, ISS)
- GPS

Unusual orbits (especially ground track): Chandra, IMAGE, CRRES (in a geosynchronous transfer orbit)

**Geostationary Graveyard orbit:**

Decommissioned geostationary satellites are sometimes “parked” about 300 km (at least) above geostationary orbit altitude in a graveyard orbit. This is done if it is not practical to keep them on orbit or bring them down to burn up.

Delta-v to reach graveyard is about 11 m/s

Delta-v to fully de-orbit is about 1500 m/s

In practice, the maneuver is difficult to complete successfully, and there is evidence that micro-meteorite impacts can still create debris that may endanger future missions.

**Changing orbit**

To boost altitude with the least effort uses a maneuver called a **Hohmann Transfer**

Added velocity $\Delta v_A$ puts the satellite into an elliptical orbit.

At apogee $\Delta v_B$ puts the satellite into a final orbit at the new altitude.

The greatest initial velocity at liftoff occurs if the launch is due east. This adds the rocket velocity to the velocity due to Earth’s rotation.
The launch site is always on a ground track, so a launch due east is into a prograde orbit with inclination the same as the launch site latitude. To reach a greater inclination, the satellite must be maneuvered out of this orbital plane ($i = 28.5^\circ$ for Cape Canaveral).

To change the inclination thrust is required in the direction perpendicular to the orbital plane.

Placing a satellite into its final orbit is usually achieved by a sequence of brief thruster bursts designed to achieve specific changes in orbital plane and altitude.

Between each burst of thrusters the satellite moves freely in a Keplerian orbit. It is tracked precisely to compute the effect of the thruster burst, and subsequent maneuvers fine-tuned to achieve the desired outcome.

To rendezvous with another satellite, an orbit in the same plane is first achieved, but at a slightly lower altitude (to catch up) or higher altitude (to be caught).


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**Measurement geometry and imaging techniques**

References: Martin, Chapter 1.6.

Satellites use several methods to form an image.

Low earth orbit satellites typically use an instrument that has a small field of view (FOV) (designed for sensor accuracy, resolution, and efficient power use), and scan the sub-satellite path to form an image.

The wider the swath of the scan the more of the earth’s surface that can be imaged in a single pass.

The satellite view is defined in terms of:

- **zenith angle** with respect to the satellite nadir

- **look angle** with respect to local vertical depends on altitude and the curvature of the earth, and differs from the zenith angle.

- **solar angle** describes the angle at which the sun illuminates the surface being observed.
If solar angle = look angle the satellite can see a direct reflection off the surface.

**Scan modes:**

A **whisk-broom** scan mode is achieved with a rotating mirror to direct the observed radiation into a fixed detector

- image is constructed from a combination of the satellite motion along its trajectory, and the rotation of the mirror
- scan lines are at an oblique angle to the ground track
- calibration can be achieved by having the scanner view a fixed internal source on each spin (good), and/or a fixed point in space for a background value
- examples: AVHRR, SeaWiFS
- multi-channel data can be received by splitting the signal after the mirror
- fixed FOV means the resolution on the ground is more coarse at last off-nadir angle

A **push-broom** scan mode uses lens to direct observing from different view angles into different detectors.

Various strategies can be devised to image multiple wavelengths with multiple detectors if this is desirable.

- Also have nadir view circular, and off-nadir view elliptical
- *dwell time* is longer than whisk broom because we don’t need a scan with a mirror (so this can improve signal/noise)
- multiple sensors can lose their calibration
- examples: Landsat Thematic Mapper, MERIS (Envisat)

**Hybrid cross-track scanner**

Linear array of sensors and rotating mirror

- increases dwell time by imaging an ellipse stretched along the trajectory direction
- still allows internal calibration on each scan
- examples: MODIS on Terra and Aqua

Other issues with off-nadir view are:
- view angle with respect to the sun affects the illumination of the surface, and the reflected vs emitted light
- scan angle alters the atmospheric path length and attenuation

Conical scan
A fixed scan angle can be achieved with a conical scan (but will still have different solar angle relative to the earth)
- The conical scan images the same point twice (looking fore and aft) which means two measurements of the same place are made through different atmospheric paths, which can be used to aid in the correction of atmospheric effects
- Example: AATSR (Advanced Along Track Scanning Radiometer)

Internet Resources on orbits and satellites:
- [http://science.nasa.gov/realtime/jtrack/Spacecraft.html](http://science.nasa.gov/realtime/jtrack/Spacecraft.html) << broken
  Shows ground tracks and 3-D orbits for all satellites
  - Click on J-Track 3D and notice the clusters of satellites in the major categories or orbits
    - Geostationary
    - Polar orbiting (NOAA, Topex, ERS, Envisat)
    - Low Earth orbit (Iridium, HST, ISS)
    - GPS
    - Unusual orbits (especially ground track)
      - Chandra
      - IMAGE
      - CRRES (in a geosynchronous transfer orbit)
- [http://heavens-above.com](http://heavens-above.com)
  Shows predicted orbits and visibility magnitudes and star charts of pass trajectories for all satellites. New Brunswick location (for homework 1) at
http://tinyurl.com/11-670-451-homework-1

- http://spaceflight.nasa.gov/realdata/elements
  - See graphics of orbital elements and data for ISS

- Real-time satellite tracking at http://www.n2yo.com