Answer Key Homework (due October 29, 2009)

Problem #1

If β is the azimuth angle of the antenna of a radar with respect to the upwind direction, the radial velocity, V_r, can be expressed as

$$V_r = V_h \cos\alpha\, cos\beta + \ w \sin\alpha$$

where V_h is the horizontal wind, α is the elevation angle of the radar antenna, and w is the vertical velocity of the target. This equation contains two unknowns and cannot be solved without an additional assumption or another measurement that determines either V_h or w. Consider a situation in which there are two identical Doppler radars separated by a few kilometers illuminating the same measurement volume using a common elevation angle.

Derive an equation for w as a function of the elevation angle and the radial velocities measured by the two radars.

Solution:

$$V_{r1} = V_h \cos \propto \cos\pi + V_v \sin\alpha$$

$$V_{r2} = V_h \cos \propto \cos(0) + V_v \sin\alpha$$

Solve simultaneously to yield

$$V_v = \frac{V_{r1} + V_{r2}}{2\sin\alpha}$$

Problem #2

The WSR-88D radar has a PRF that varies between 318 s^{-1} and 1304 s^{-1}. Assuming that NFFT=128 and that NCOH=1, compute the unambiguous range in kilometers, unambiguous velocity in meters per second, and velocity resolution in meters per second for these two values of the PRF. Comment on the likely application of these two PRFs (one or two sentences).

Solution:

As discussed in class, the wavelength, λ, of the WSR-88D is 10 cm. Let the Long Pulse (318 s^{-1}) be denoted as PRP_{LP} and the Short Pulse (1304 s^{-1}) be denoted as PRP_{SP}.

The Pulse Repetition Period (PRP) is the reciprocal of the Pulse Repetition Frequency (PRF), so

$$PRP_{LP} = \frac{1}{318\, s^{-1}} = 3.14 \times 10^{-3}\, s$$

$$PRP_{SP} = \frac{1}{1304\, s^{-1}} = 7.66 \times 10^{-4}\, s$$

The Unambiguous Range, R_a, for the two pulses is:

$$R_{a-LP} = \frac{c\, PRP_{LP}}{2} = \frac{(3 \times 10^{8}\, ms^{-1})\, 3.14 \times 10^{-3}\, s}{2}$$

$$= 4.71 \times 10^{5} m = 471\, km$$

$$R_{a-SP} = \frac{c\, PRP_{SP}}{2} = \frac{(3 \times 10^{8}\, ms^{-1})\, 7.66 \times 10^{-4}\, s}{2}$$

$$= 1.15 \times 10^{5} m = 115\, km$$

The Unambiguous Velocity, V_a, for the two pulses is:

$$V_{a-LP} = \frac{\lambda}{4\,(NCOH)\,(PRP_{LP})} = \frac{10\, cm}{4\,(1)\, 3.14 \times 10^{-3}\, s} \cong 8.0 \times 10^{2}\, cm\, s^{-1} \cong 8\, m\, s^{-1}$$

$$V_{a-SP} = \frac{\lambda}{4\,(NCOH)\,(PRP_{SP})} = \frac{10\, cm}{4\,(1)\, 7.66 \times 10^{-4}\, s} \cong 3.3 \times 10^{3}\, cm\, s^{-1} \cong 33\, m\, s^{-1}$$

The Velocity Resolution, ΔV, for the two pulses is:

$$\Delta V_{LP} = \frac{\lambda}{2(NFFT)\,(NCOH)\,(PRP_{LP})} = \frac{10\, cm}{2\,(128)(1)\, 3.14 \times 10^{-3}\, s} \cong 12\, cm\, s^{-1} \cong 0.12\, m\, s^{-1}$$

$$\Delta V_{SP} = \frac{\lambda}{2(NFFT)\,(NCOH)\,(PRP_{SP})} = \frac{10\, cm}{2\,(128)(1)\, 7.66 \times 10^{-4}\, s} \cong 51\, cm\, s^{-1} \cong 0.5\, m\, s^{-1}$$

The Long Pulse has and insufficient Unambiguous Velocity to cover the expected range of motions in the sample volume and is, therefore, the preferred mode for reflectivity measurements due to its larger Unambiguous Range relative to the Short Pulse. The Short Pulse has sufficient Unambiguous Velocity to handle the range of observed motion in most cases, though an even shorter pulse would be required to resolve some tornados.

Problem #3 (Graduate Students)

(1) The size distribution of raindrops is often represented by a Marshall-Palmer (1948) distribution, which has the form

$$N_r = N_0 \exp(-2\Lambda r)$$

where r is the radius of the raindrop, $\Lambda = CR^{-0.21}(cm^{-1}), C = 41, R = Rainfall\ Rate\ in\ \ mm\ hr^{-1}$, and N_0 is a constant that has been determined to be 0.08 cm^{-4}. Suppose that a Doppler radar is capable of measuring the radial velocity in the zenith direction (elevation angle of 90 degrees) or there are two Doppler radars available so that the vertical velocity of the raindrops can be measured. Note that the terminal fall velocity of small raindrops is given by $w_{terminal-raindrop} = kr^{1/2}$, where $k \approx 2.01 \times 10^3\ cm^{1/2}\ s^{-1}$ and r is the droplet radius. Derive an equation for the mean velocity of the population of raindrops as a function of $k, C, and\ R$.

(b) If the actual rainfall rate is 5 mm hr^{-1}, how much uncertainty is to be expected in the radar estimate of the rainfall rate for a zenith pointing WSR-88D (if it were possible to look toward zenith) if the mean velocity is estimated by the radar signal processor, which is typically the case.

Solution:

$$v_r = kr^{1/2} \quad \rightarrow \quad r = \left(\frac{v_r}{k}\right)^2 \quad \rightarrow \quad dr = d\left(\frac{v_r}{k}\right)^2 = \frac{1}{k^2} dv_r^2 = \frac{2}{k^2}\ v_r dv_r \qquad (1)$$

Transforming the Marshall-Palmer distribution to velocity space

$$dN_v = n_0 \exp\left(-2\Lambda \frac{v_r^2}{k^2}\right) dr \qquad (2)$$

Combining (1) and (2) yields

$$dN_{v_r} = \frac{2n_0}{k^2} \mathrm{v_r} \exp\left(-2\Lambda \frac{v_r^2}{k^2}\right) dv_r \qquad (3)$$

The average velocity is written as

$$\overline{v_r} = \frac{\int_0^\infty v_r dN_{v_r}}{N} \tag{4}$$

Integrating the numerator of (4)

$$\int_0^\infty v_r dN_{v_r} = \int_0^\infty \frac{2n_0}{k^2} \mathrm{v_r} \exp\left(-2\Lambda \frac{v_r^2}{k^2}\right) dv_r \tag{5}$$

$$\int_0^\infty \frac{2n_0}{k^2} \mathrm{v_r} \exp\left(-2\Lambda \frac{v_r^2}{k^2}\right) dv_r = n_0 \frac{k}{4\Lambda} \left(\frac{\pi}{2\Lambda}\right)^{1/2} \tag{6}$$

Integrating the denominator of (4)

$$N = \int_0^\infty n_0 \exp(-2\Lambda r)\, dr = \frac{n_0}{2\Lambda} \tag{7}$$

Combining (4), (6), (7), and the definition of Λ

$$\overline{v_r} = \frac{k}{2}\left(\frac{\pi}{2\Lambda}\right)^{1/2} = \frac{k}{2}\left(\frac{\pi}{2C}\right)^{1/2} R^{0.105} \tag{8***}$$