

## Remote Sensing Homework due September 24—Revised Version

(1) The Equivalent Earth Propagation Model is given as

$$h = \left[ r^2 + \left( \frac{4}{3} a \right)^2 + 2r \left( \frac{4}{3} a \right) \sin \theta_e \right]^{1/2} - \frac{4}{3} a$$

and

$$s = \frac{4}{3} a \sin^{-1} \left( \frac{r \cos \theta_e}{\frac{4}{3} a + h} \right)$$

where  $h$  is the height above the surface at some arc distance,  $s$ , as shown in class,  $a$  is the Earth's radius (kilometers),  $r$  is the range to the target,  $\theta_e$  is the elevation angle of the radar antenna. Compute and plot the height above the surface for a pulse transmitted with elevation angle of  $\theta_e = 0.5^\circ$ .

(2) Ground-based propagation ducts may be present when there is a strong temperature inversion and the Earth's surface. Such a situation may alter the curvature of the propagation path of the radar beam so that it is directed toward the Earth's surface, as shown in class. It can be shown that there is a critical elevation angle known as the penetration angle,  $\theta_p$ , below which redirection toward the surface is guaranteed. This critical angle is given by

$$\theta_p = \sin^{-1} \left[ \frac{-2h_t(1 + \beta_0 a)}{a} \right]^{1/2}$$

where  $\beta_0 = -3.0 \times 10^{-4} \text{ km}^{-1}$  is related to the strength of the inversion at the surface (quantifies the lapse rate of refractivity,  $N$ ) and  $h_t$  is the height at the top of the surface based inversion.

(a) Compute  $\theta_p$  for an inversions that are 100 and 200 meters in depth

(b) Assume that the strength of the inversion is degraded to  $\beta_0 = -2.0 \times 10^{-4} \text{ km}^{-1}$  and repeat the calculation in part a.

A linear propagation model for a ground based inversion yields the height and the arc distance for the maximum elevation attained by a propagating beam, which are given by

$$h_r = \frac{-a \sin^2 \theta_e}{2(\beta_0 a + 1)}$$

and

$$s_r = \frac{-a \sin \theta_e \cos \theta_e}{\beta_0 a + 1}$$

(c) Compute the height above the surface for a ground based inversion when for a pulse transmitted with elevation angle of  $\theta_e = 0.2^\circ$  using  $\beta_0 = -3.0 \times 10^{-4} \text{ km}^{-1}$ .

(3) Compute the radar effective reflectivity factor,  $Z$ , in  $\text{mm}^6 \text{ m}^{-3}$  for a cloud containing  $200 \text{ cm}^{-3}$  droplets that have a radius of  $10 \mu\text{m}$ . Convert this value to dBZ.

(4) GRADUATE STUDENTS: The radar equation is given by

$$dBZ = 10 \log_{10}[P_r(mW)] + 20 \log_{10}[r(km)] + 10 \log_{10} R_c$$

If the radar constant,  $R_c = 65.6 \text{ dB}$  and the system noise power is  $10 \log_{10}[P_r(mW)] = 113 \text{ dBm}$  (note that dBm is decibels referenced to a millivolt and dBm and its value may be directly substituted into the radar equation). The minimum detectable reflectivity is the reflectivity necessary for the echo to be detected in the presence of noise. Compute the minimum detectable reflectivity in dBZ for this radar at a range of 5 km.

