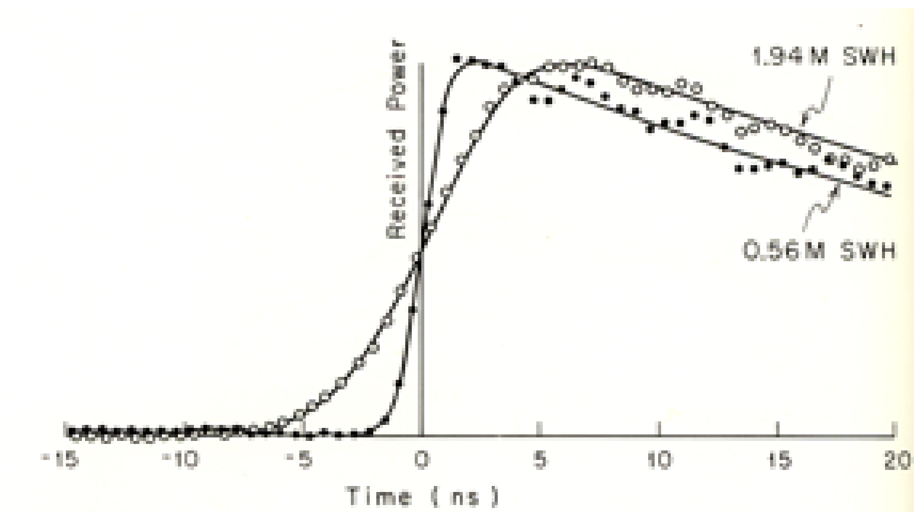
Homework

September 29, 2009

Problem 1 (John Wilkin)

Explain why the Significant Wave Height of wind waves on the sea surface alters the shape of the altimeter radar reflections shown in the figure below.



Problem 2 (John Wilkin)

What properties of the ionosphere and troposphere are observed or analyzed in order to make corrections to altimeter radar travel times? How is this information obtained?

Problem 3 (Ben Kravitz)

Consider a section of the New Jersey coastline with two CODAR stations, one operating at a transmission frequency of 5 MHz and one operating at 25 MHz.

The 5 MHz station is 40 km due North of the 25 MHz station. Both stations are observing a collection of waves at sea with the goal of determining the magnitude and direction of the surface current at that point. The collection of waves is 30 km from the 25 MHz station and 50 km from the 5 MHz station.

a. Draw a picture of the setup. (Hint: it should look like a triangle.) Include both CODAR stations and the collection of waves they are observing. Also include values for the angles of the triangle. (Hint: The Pythagorean Theorem should help you determine at least one of these angles.)

b. Each of the CODAR stations transmits at its specified frequency. The wave it sends out is scattered off the sea surface and returns to the CODAR station. Using the formula $c = f_r \lambda_r$, where c is the speed of light (3×10^8 m/s), f_r is the transmitted frequency, and λ_r is the wavelength of the transmitted signal, determine the wavelength of the transmitted signal for each station. Also, using the formula $\lambda_r = 2\lambda_s$, where λ_s is the wavelength of the sea surface wave, determine the wavelengths of the sea surface waves that can be measured by the CODAR stations.

c. When the signal returns to the CODAR station, it may return at a slightly different frequency than at which it was transmitted. This is indicative of a Doppler shift, and the corresponding equation is

$$f_D = \frac{2V}{\lambda_r}$$

where f_D is the Doppler shift frequency, V is the radial velocity of the sea surface wave, and λ_r is the wavelength of the transmitted wave. The 25 MHz CODAR station receives a return signal of 24,999,999.43 Hz, and the 5 MHz CODAR station receives a return signal of 4,999,999.76 Hz. Calculate the radial velocities for each of the stations and tell whether the wave is moving toward or away from that station.

d. The radial velocity measured is equal to the sum of the wave speed and the surface current velocity. Using the dispersion relation

$$c = \sqrt{\frac{g\lambda_s}{2\pi} \tanh\left(\frac{2\pi h}{\lambda_s}\right)}$$

calculate the wave speed relevant to each station. Assume $g = 9.81$ m/s^2, $h = 100$ m is the depth of the water, and use the formula

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Then calculate the velocity for the surface current (in the radial direction) as measured by each station. Be sure to indicate which way the surface current is flowing. If we've chosen h to be 100 m, where in the ocean are we? Are the values we've obtained for surface current reasonable for this part of the ocean?

e. **[EXTRA CREDIT]** Using the results you obtained from both stations, calculate the resultant surface current velocity and direction.

Problem 4 (Mark Miller)

The Doppler frequency shift of a single target illuminated by a radar beam is given by

$$f = \frac{2v_r}{\lambda}$$

where v_r is the radial velocity (velocity toward or away from the radar antenna) and λ is the radar wavelength. The volume sampled by a radar pulse typically contains a large population of droplets moving with a wide range of different velocities modulated by turbulent motions within the radar sampling volume. The receiver electronics often used to measure the Doppler frequency shift in a radar measures and records the mean Doppler shift frequency $\overline{f}$, which is related to the mean radial velocity $\overline{v_r}$. These means are related through

$$\overline{f} = \frac{2\overline{v_r}}{\lambda}$$

Consider a situation in which the number of droplets N_{v_r} with radial velocities falling in a specified narrow range of radial velocities dN_{v_r} is given by

$$dN_{v_r} = N_{v_r} \exp\left(-\beta^2 v_r^2\right) dv_r$$

where dN_{v_r} is an increment in the number of particles with a specified radial velocity and β is a constant. The mean velocity is given by

$$\overline{v_r} = \frac{\int_0^\infty v_r dN_{v_r}}{N}$$

The Root Mean Square (RMS) of a quantity is often measured by devices, including a radar receiver, that are attempting to characterize a time varying signal. It is given by

$$x_{\mathrm{rms}} = \sqrt{\overline{x}^2}$$

Compute $\overline{v_r}$, $\overline{f}$, and f_{rms} as a function of β and λ. Does a measurement of f_{rms} enable a determination of $\overline{v_r}$ if the velocities of the targets are distributed according to the equation above?