Lecture Notes **Prepared by Prof. J. Francis Spring 2005**

Orbits and Navigation

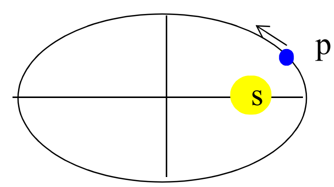
Material from KVH Chapters 2 and 4. Figure numbers below from KVH.

Motivation -- Why study orbits?

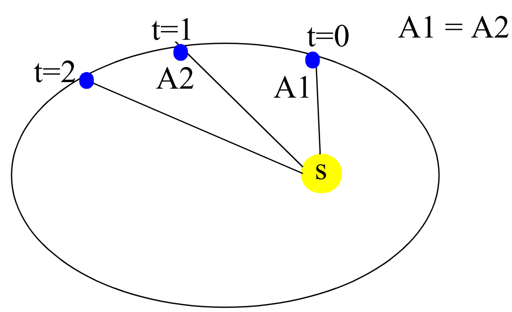
1. Defines spatial and temporal resolution
2. Understand trade-offs between satellite design and sampling
3. Understand accuracy of location, data errors, orbit trajectories
   e.g., need certain orbits to look at poles, tropics, whole globe, same area continually

Kepler's Laws (50 years before Newton)

1. Planets move in elliptical orbits with the sun at one focus



2. Radius vector from sun to planet sweeps out equal areas in equal time



3. $P^2 \propto \bar{r}^3$

P = orbital period
$\bar{r}$ = mean radius

$\Rightarrow P_{Pluto} >> P_{Mercury}$

Try this excellent interactive website to demonstrate Kepler's laws:
http://www.physics.sjsu.edu/tomley/kepler.html

Newton's Laws

1. Force (F) required to change velocity (speed and/or direction) of body

2.

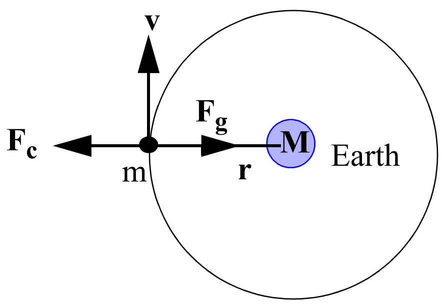
$$F = ma = m\frac{dv}{dt}$$

m = mass [kg]
v = velocity [m s^{-1}]
F = force [kg m s^{-2} N

t = time [s]
a = acceleration [m s^{-2}]

3.Newton's Law of Universal Gravitation

What are the forces?



$\mathbf{F}_g = \mathbf{F}_c$

$\mathbf{F}_g$ = gravitational force (centripetal: center-seeking)

$\mathbf{F}_c$ = centrifugal (center-fleeing)

Force of attraction between 2 point masses M, m separated by distance r is

$$\boldsymbol{F}_g = \frac{GMm}{r^2}$$

G = universal gravitational constant = 6.67E-11 $Nm^2\ kg^{-2}$

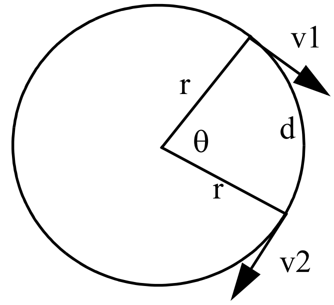
$$\boldsymbol{F}_c = \frac{mv^2}{r} \text{ (uniform circular motion)}$$

Review: Centripetal acceleration

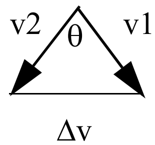
$F_c = ma = m\frac{\Delta v}{\Delta t}$

distance = rate x time

$d = v\Delta t$



Take velocity vectors and do graphical vector subtraction:



(v1 ⊥ r and v2 ⊥ r so angle between is the same)

|v1| = |v2| so

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = a = \frac{v^2}{r}$$

$$\boldsymbol{F}_c = \frac{mv^2}{r}$$

So if $\mathbf{F}_c = \mathbf{F}_g$ then $v^2 = \frac{GM}{r}$ which says that the *orbit does not depend on m*!

To get orbit period T (time for one revolution):

$C = vT$ (distance = rate x time; C = circumference)

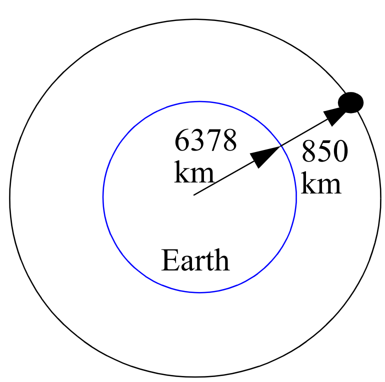
$$v = \frac{C}{t} = \frac{2\pi r}{T}$$

$$\frac{(2\pi r)^2}{T^2} = \frac{GM}{r}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

E.g.:

Current NOAA polar orbiters are at altitudes ~850 km. How long does it take to complete 1 orbit?



$r = r_s + r_e$

= 850 km + 6378 km

M_e = 6E24 kg

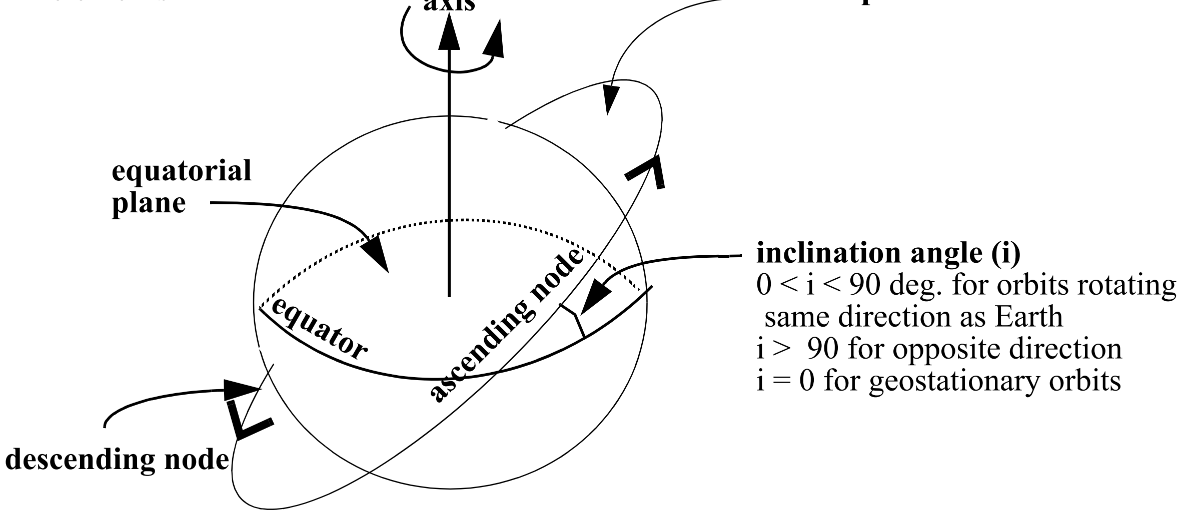
$$T^2 = \frac{4\pi^2\left(7320km\left(\frac{10^3 m}{km}\right)\right)^3}{GMe}$$

T ~ 102 min

Fig. 2.8, p. 24 KVH

Geosynchronous orbit: Satellite orbit that circles Earth once per day. To be geostationary, i.e., satellite stays over same location on Earth, orbit must also be directly above equator.

Orbital elements



Prograde (retrograde) orbit: equator crossing progresses from west to east (east to west) with each orbit. Prograde orbits are more common.

Keplerian orbits: orbital plane is fixed while Earth rotates around the sun.

Satellite passes at different times of day

Hard to fit data arrival into schedules

Hard to orient solar panels

Sun-synchronous orbit: Inclination angle chosen so orbit retrogrades (shifts equator crossings in opposite direction to Earth's rotation) at exact rate as Earth's orbit around sun -- 2π radians in 365.25 days ~ 1 degree of longitude per day.

Fig 2.7 KVH

Sources of orbit perturbations

Non-spherical, non-homogeneous Earth

Gravitational force by other objects (moon, sun)

Radiation pressure

Solar wind

Friction with particles

Electromagnetic interactions with Earth's magnetic field