Lecture Notes **Prepared by Mike Foster Spring 2007**

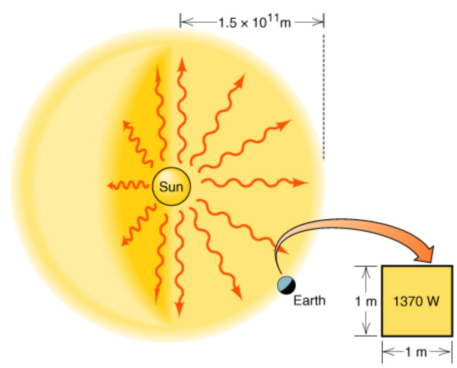
Solar Radiation

Sources:
K. N. Liou (2002) *An Introduction to Atmospheric Radiation*, Chapter 1, 2
S. Q. Kidder & T. H. Vander Haar (1995) *Satellite Meteorology: An Introduction*, Chapter 3

The Sun

Nearly all of the energy that the earth receives and that drives the ocean and atmosphere comes from the sun. The distance between the earth and sun averages about 150 million kilometers (93 million miles). The great distance between the earth and sun, plus the earth's relatively small size compared to the sun allows us to assume that the sun's rays strike the earth in straight paths (irradiance vs. radiance).

The solar radiation reaching the earth originates (for our purposes) from this layer of the sun called the photosphere, which coincides with the visible disk of the sun. Although the sun is made up of gas, the photosphere is referred to as the surface of the sun. The photosphere is a relatively thin layer, about 450 km thick. The temperature of the photosphere varies from 8000 K in the lower layer to about 4000 K in the upper layer, but the effective temperature when calculating the Planck curve is about 5800 K.



A fundamental property of electromagnetic radiation is that it can transport energy. Therefore the units used to measure electromagnetic radiation are based on energy units.
Radiant Energy is the basic energy unit and is measured by the Joule (J).
Radiant Flux is radiant energy per unit time: J s-1 = Watt (W).
Radiant Flux Density is the radiant flux energy over a unit area: W m^{-2}.
Radiant flux density is a commonly used term, and as such it is divided into two terms:

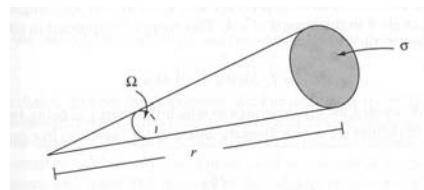
- **Irradiance** (E) is the radiant flux density incident in a surface.
- **Radiant Exitance** (M) is the radiant flux density emerging from a surface.

Finally, when we're talking about the measurements taken by satellites, we need to take into account directionality:
Radiance (L) is the radiant flux density per unit solid angle, which is the measurement generally taken by satellites.

The total amount of energy reaching the top of the atmosphere is called the **solar constant**. It is defined as the flux of solar energy (per unit time) across a surface of unit area normal to the solar beam at the mean distance between the sun and the earth. The sun emits energy at the rate of 6.2×10^7 W m^{-2} isotropically. The total amount of energy reaching the top of the atmosphere varies slightly but is about 1370 W m^{-2}.

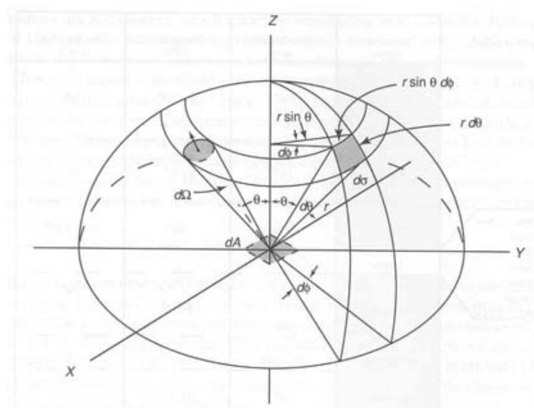
Radiation is a vector quantity, meaning that it has a direction. The **solid angle**, omega Ω, is used to describe this directional dependence by considering the amount of radiation confined to a prescribed area. **Solid angle** is defined as the ratio of the area ,sigma σ, of a spherical surface intercepted at the core to the square of the radius *r*:

$$\Omega=\sigma/r2$$



Units of solid angle are normally expressed in steradians (sr). For a sphere whose surface area is **4πr2**, the solid angle is **4π sr**. If we want a differential solid angle, we need to define solid angle in terms of polar coordinates. We construct a sphere whose central point is denoted as *O*. Assume a line through point *O* moving in space and intersecting an arbitrary surface some distance *r* from point *O*. First, we define:
Azimuthal angle, phi φ: the horizontal direction of a vector, measured in terms of a 360 degree compass with north being 0 degrees.
Zenith angle, theta θ: the angle measured from the normal (vertical) to the surface.
The differential area in polar coordinates should be:

$$d\sigma=(rd\theta)(r(\sin\theta)d\varphi$$



So the differential solid angle should be:

$$d\Omega = d\sigma / r2 = (\sin\theta)d\theta d\varphi = d\mu d\varphi$$

Where $\mu = \cos\theta$.

It is useful for remote sensing to know what the sun angle is at a given location for a given time. To this end we can define the solar position using the following terminology:
Solar declination – the latitude at which the sun is directly overhead at solar noon,
Zenith angle – the angle between a point directly overhead and the sun at solar noon,
Solar elevation (sun) angle – the angle of the sun above the horizon at solar noon.
To calculate the noon zenith angle, simply find the number of degrees of latitude separating the location receiving the direct vertical rays of the sun and location in question. The solar elevation angle is calculated by subtracting the zenith angle from 90 degrees. The solar declination angle can be approximated using the following equation: 23.5*sin(N), where N is the number of days to the closest equinox.

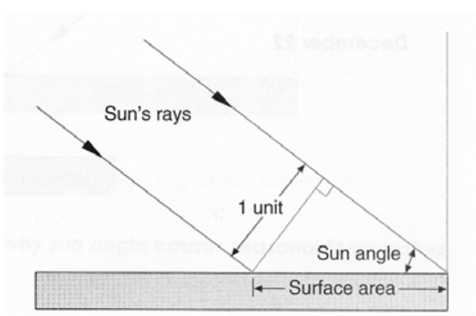


Figure 2-6b

The sun elevation angle is important because it affects the intensity of solar radiation received at the earth's surface. When the sun angle is large, the solar rays are more direct, are spread over a smaller surface area, and pass through less atmosphere, resulting in greater radiation per unit area. If a beam of sunshine strikes the flat earth at a sun angle of 36 ½° above the horizon, the beam is actually spread out over 1.681 area units, which diminishes the maximum intensity of radiation to ~60%. For example, in the morning and evening, the amount of solar energy received per unit area is much less than at noon. This is why you are often warned to reduce sun exposure during mid-day hours.

Radiative Processes in the Atmosphere

If we solve Planck's function solves for the radiation emission for a blackbody, when integrated over all wavelengths we get:

$$B_\lambda(T)=\sigma T^4$$

This result is called the *Stefan-Boltzmann law* and σ=(5.67051 x 10-8 W m^{-2} K^{-4}) is the *Stefan-Boltzmann constant*. There is an approximation to the Planck function that can be useful at millimeter and centimeter wavelengths, and for temperature like those encountered on the earth and in its atmosphere. This approximation is called *Rayleigh-Jeans approximation:*

$$B_\lambda(T)=\frac{c_1\lambda^{-5}}{\exp(\frac{c_2}{\lambda T})-1}\approx\frac{c_1}{c_2}\lambda^{-4}T$$

The two constants, c_1 and c_2, are called the first and second radiation constants (1.19104 x 10^{-16} W m^2 sr^{-1}, 1.4387 x 10^{-2} m K). This approximation says that in the microwave region of the EM spectrum, the radiance is proportional to the temperature. In remote sensing, it is often customary to divide the radiance by $(c_1/c_2)\,\lambda^{-4}$ to yield a *brightness temperature*.

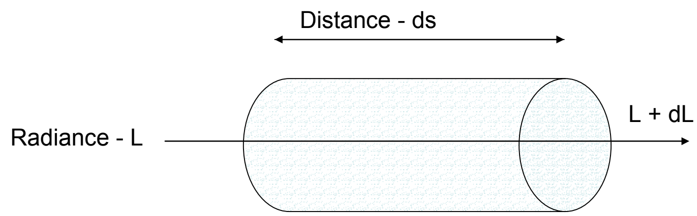
Remember that a blackbody absorbs the maximum amount of radiation, and therefore must emit the maximum amount. The emissivity of a given wavelength, ε_λ, is the ratio of the emitting intensity to the Planck function, and is equal to the absorptivity, α_λ, for the medium under thermodynamic equilibrium; this is called *Kirchoff's law*.

A gray body is characterized by incomplete absorption and emission, $\alpha_\lambda = \varepsilon_\lambda < 1$.

The *index of refraction*, *n*, of a substance is the ratio of the speed of light in a vacuum to the speed that EM radiation travels in that substance. At sea level, the index of refraction is approximately 1.0003. For most purposes, the speed of light in a vacuum can be used safely in the atmosphere. However, strong vertical gradients of atmospheric density and humidity can result in strong vertical gradient of *n*. This causes bending of EM rays and slight dislocation of satellite scan spots.

Now we can discuss the transfer of solar radiation through the atmosphere. Let us consider radiation incident on a differential volume of atmosphere. Disregarding polarization, there are four things that can occur to the radiation beam as it travels the volume of atmosphere (See figure).

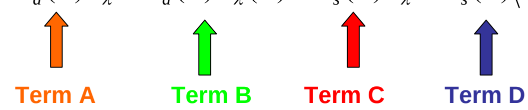
σ_a is the volume absorption coefficient, L is the radiance, and σ_s is the scattering coefficient. B is the Planckian emission, but from Kirchhoff's law we know a material is as good an emitter as an absorber, so the absorption coefficient can also be used as a sort of emission coefficient. The equation at the bottom of this figure is a simplified form of the *radiative transfer equation*. For now, it should be enough to understand what terms go into the radiative transfer equation.



A. Radiation from the beam can be absorbed by the material (depletion term)
B. Radiation can be emitted by the material (source term)
C. Radiation from the beam can be scattered into other directions (depletion term)
D. Radiation from other directions can be scattered into the beam (source term)

$$\frac{dL_\lambda}{ds} = A + B + C + D$$

$$\frac{dL_\lambda}{ds} = -\sigma_a(\lambda)L_\lambda + \sigma_a(\lambda)B_\lambda(T) - \sigma_s(\lambda)L_\lambda + \sigma_s(\lambda)\langle L'_\lambda \rangle$$



Scattering

Radiation scattered from a particle is a function of several things: particle shape, particle size, particle index of refraction, wavelength of radiation and viewing geometry. In 1908, Mie applied Maxwell's equations to the case of a plane electromagnetic wave incident on a sphere. Mie showed that for a spherical scatterer, the scattered radiation is a function of only viewing angle, index of refraction and the *size parameter*. The size parameter is defined as:

$$\chi = 2\pi r/\lambda$$

Where *r* is the radius of the sphere. With the size parameter, we can separate scattering into three regimes: Mie scattering, geometric optics and Rayleigh scattering.

For size parameters in the range 0.1-50, the wavelength of the radiation and the circumference of the particle are comparable. Radiation strongly interacts with the particle, and therefore the full Mie equations must be used. These equations are important because they have been applied extensively to the detection of raindrops by radar, the study of aerosols, and the interaction of cloud droplets with IR radiation.

The intensity scattered by a particle as a function of direction can be given by:

$$I(\Theta) = I_0\left(\frac{\sigma_s}{r^2}\right)\frac{P(\Theta)}{4\pi}$$

I_0 is the incident intensity, P is the scattering phase function, σ_s is the scattering cross section, and 4π is the solid angle for the entire spherical space.

We're not going to go into a full discussion of Mie scattering as the equations quickly become complex, but there are a couple of points to make here:

The *scattering phase function* determines in which direction the radiation is scattered. This function depends a lot on the size parameter. As the size parameter increases, the phase function becomes strongly peaked in the forward direction.

We can solve the scattering cross section in terms of the *scattering efficiency*, which is a useful way to discuss scattering in the atmosphere. The scattering efficiency, Q_s, is the ratio of the total scattered radiation to the incident radiation (regardless of direction).

In clouds, there is usually a distribution of droplet sizes. If the scatterers are sufficiently far apart (many wavelengths) that they act independently, we can define the scattering coefficient in terms of the scattering efficiency and the droplet size distribution:

$$\sigma_s(\lambda) = \int_0^\infty \pi r^2 Q_s N(r) dr \, .$$

As mentioned before, many substances absorb radiation as well as scatter it. This can be conveniently taken into account by letting the *index of refraction* be represented by a complex number: $m \equiv n - in'$ where n is the real part of the index of refraction as defined above, and n' is the imaginary part that accounts for absorption inside the scatterers.

For χ less than ~0.1, the particle is small in comparison with the wavelength of radiation. Because of this, the *Rayleigh scattering* is insensitive to particle shape.

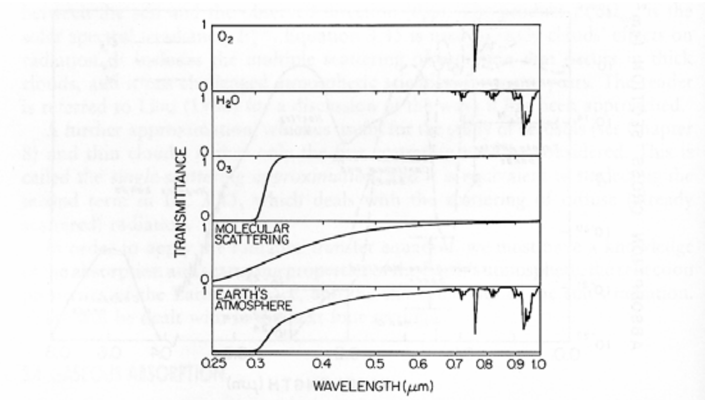


FIGURE 3.11. Vertical transmittance of the Earth's atmosphere between 0.25 and 1.0 μm. Rayleigh scattering by air molecules is the chief limitation to the transfer of visible radiation through the clear atmosphere. The effects of aerosols have not been included in these curves. [Calculated using LOWTRAN 6 (Kneizys *et al.*, 1983).]

Figure 3.11 (KVH)

Figure 3.11 shows the vertical transmittance of the Earth's atmosphere due to Rayleigh (molecular) scattering. At visible and uv wavelengths, Rayleigh scattering by air molecules must be taken into account. Aerosol optical depth can be measured over the ocean if the observed radiance is corrected for Rayleigh scattering by air molecules. Rayleigh scattering of air molecules must also be taken into account when making uv measurements of ozone.

Rayleigh particles scattering equally well in the forward and backward directions. Rayleigh particles, for example cloud droplets, can also absorb radiation. Note that in the Rayleigh regime, the absorption efficiency is proportional to the droplet radius. So the absorption coefficient is proportional to the total volume of scatterer per unit volume of atmosphere. For example, in a cloud absorption due to Rayleigh-size cloud droplets is proportional to the liquid water content.

For χ greater than about 50, the sphere is large in comparison with the wavelength of radiation. This is the realm of *geometric optics*, where rays, which are reflected and refracted at the surface of a scatterer, can be traced. The interaction of solar radiation with virtually all types of hydrometeors falls in this regime. A wide variety of optical phenomena such as rainbows and halos can be explained with geometric optics. The interaction of infrared radiation with precipitation-size particles also falls within this regime.

In the geometric optics regime, the radiation that strikes the drop is scattered and refracted in equal amounts.

Clouds

Clouds play a large role in satellite meteorology, and so it is important to understand in a general way the interaction of clouds with radiation. Cloud consists of water drops or ice crystals with radii on the order of 10 μm. Drops with radii of 100 μm have significant fall speed and constitute drizzle. Drops with radii of 1000 μm (1 mm) are raindrops. Clouds have drop concentrations on the order of 10^8 m^{-3}. That means that drops are on the order of 1 mm apart.

In the visible portion of the spectrum (λ~0.5 μm), cloud drops are geometric scatterers. Therefore, the scattering efficiency is approximately 2. The scattering coefficient, then, is ~0.1 m^{-1}. A cloud only a few tens of meters thick is sufficient to scatter all of the visible radiation incident on it. Since liquid water does not absorb visible radiation well, very little of the radiation is absorbed. Most radiation emerges from the cloud somewhere after being scattered many times. Since clouds have a distribution of drop sizes and the size parameter is large, all visible wavelengths are scattered nearly equal well. Clouds, therefore, appear white.

Cirrus clouds have a higher transmittance than water clouds because ice clouds have far fewer particles per unit volume than water clouds and because water is a better absorber than ice. Cirrus clouds can also be vertically thinner than water clouds. In general thin cirrus clouds are difficult to detect with satellite radiometers. Thin cirrus can cause problems in the retrieval of atmospheric sounding.

Surface Reflection

Reflected radiation, particularly reflected solar and microwave radiation, is very important to satellite meteorology. Several quantities are used to describe reflected radiation. The most basic is the bi-directional reflectance, which is related to the fraction of radiation from incident direction to a known reflected direction. Probably the most frequently used reflection quantity is the albedo, which is the ratio of the radiant exitance to irradiance. Albedo is a unitless ratio between zero and one. It is a function of neither incoming nor outgoing angles, however this does not mean that the albedo is constant. If the incoming radiation changes, the albedo will change.

There are many types of reflecting surfaces. Two limiting cases are of interest here. The first is a Lambertian surface, or isotropic reflector that reflects radiation uniformly in all direction. Flat white paint approximates a perfect *Lambertian reflector*. A *specular reflector* is like a mirror. The bi-directional reflectance is strongly peaked. Solar radiation from a perfect specular reflector would be observed only at the zenith angle equal to the solar zenith angle and at the azimuth angle equal to the solar azimuth angle plus 180°. Water surfaces are similar to specular reflectors, except that real water surfaces are somewhat rough, so the solar reflection is blurred and larger than the sun. This is called *sun glint* or *sun glitter*.

One other reflectance term that is important is the *anisotropic reflectance* factor, which gives unequal distribution of reflectance to various angles.