Solar Radiation

Sources:
K. N. Liou (2002) *An Introduction to Atmospheric Radiation*, Chapter 1, 2
S. Q. Kidder & T. H. Vander Haar (1995) *Satellite Meteorology: An Introduction*, Chapter 3
G. Carbone (2001) *Exercises for "Weather and Climate, 4th Edition"*, Lab 2

All of the information received by a satellite about the earth and its atmosphere comes in the form of electromagnetic radiation. Therefore it is important to understand the mechanisms by which this radiation is generated and how it interacts with the atmosphere.

1. Basics of Radiative Transfer

Electromagnetic Spectrum

The *electromagnetic spectrum* is composed of visible light, gamma rays, x-rays, ultraviolet light, infrared radiation, microwaves, television signals and radio waves. All electronic magnetic (EM) waves travel at the speed of light, c=2.99792458x10^{8} m s^{-1} and are composed of alternating electrioc and magnetic fields. EM radiation is often specified by its wavelength, λ. The conventional unit for wavelength is microns (μm) where 1 μm = 10^{-4} cm = 10^{-6} m. Ultraviolet radiation is sometimes described in nanometers (nm = 10^{-9} m).

An alternative way of describing radiation is to give its *frequency*, ν. The fundamental unit of frequency is the hertz (Hz), or one cycle per second. The frequency can be related to the wavelength using the following formula: $\nu = \frac{c}{\lambda}$.

The human eye is sensitive to electromagnetic waves with frequencies between 4.3-7.5 x10^{14} Hz (cps). This corresponds to red to violet. The eye is not sensitive to frequencies above 7.5x10^{14} Hz (ultraviolet) or to frequencies below 4.3 x 10^{14} Hz. Infrared radiation is an important component of the global radiation budget, but the human eye cannot detect it. Infrared radiation occurs in wavelengths between 3x10^{12} and 4.3 x 10^{14} Hz. Microwaves cover the frequencies between 3 x 10^{10} Hz to 3 x10^{12} Hz. Microwave radiation is often described in frequency units of GHz.

For infrared radiation, it is more customary to use *wavenumbers*, κ, to describe electromagnetic radiation. A wavenumber is the reciprocal of the wavelength, $\kappa = \frac{1}{\lambda}$. Traditionally, wavenumber is expressed in units of inverse centimeters (cm^{-1}). For example, radiation with a 15-μm wavelength has a 667 cm^{-1} wavenumber. Since wavenumber is inversely proportional to wavelength, it is directly proportional to frequency.

A fundamental property of EM radiation is that it can transport energy. (Note that a table of symbols can be found on p. 49 in KVH.) So many of the units used to quantify EM radiation are based on energy. The basic unit of *radiant energy* is the joule. *Radiant flux* is the radiant energy per unit time, measured in watts (W). *Radiant flux density* is the radiant flux crossing a unit area, and is measured in watts per square meter (W m^{-2}). *Irradiance* (*E*) is the radiant flux density incident on an area.

Radiation is a vector quantity, meaning that it has a direction. The *solid angle*, *Ω*, is used to describe this directional dependence by considering the amount of radiation confined to a prescribed area. Solid angle is defined as the ratio of the area σ of a spherical surface intercepted at the core to the square of the radius *r*: $\Omega = \frac{\sigma}{r^2}$.

Units of solid angle are normally expressed in steradians (sr). For a sphere whose surface area is $4\pi r^2$, the solid angle is 4π sr. If we want a differential solid angle, we need to define solid angle in terms of polar coordinates. We construct a sphere whose central point is denoted as *O*. Assume a line through point *O* moving in space and intersecting an arbitrary surface some distance *r* from point *O*. The differential area in polar coordinates should be, $d\sigma = (rd\theta)(r(\sin\theta)d\phi)$.

So the differential solid angle will be $d\Omega = \frac{d\sigma}{r^2} = (\sin\theta)d\theta d\phi = d\mu d\phi$ where $\mu \equiv \cos\theta$.

Radiant flux density per unit solid angle is known as *radiance*, *L*. *Radiance exitance* is the total amount of radiation leaving a surface. To calculate the radiance exitance, we must integrate the radiance over the 2π sr above the surface. Radiance indicated the radiation passing through an area perpendicular to the beam. For other directions, we must weight the radiance by cos*θ*. Therefore the radiance exitance is, $M = \int_0^{2\pi} L(\theta,\phi)\cos\theta d\Omega = \int_0^{2\pi}\int_0^{\pi/2} L(\theta,\phi)\cos\theta\sin\theta d\theta d\varphi = \int_0^{2\pi}\int_0^1 L(\mu,\phi)\mu d\mu d\phi$.

If the radiance is independent of direction (isotropic), then we can move *L* outside of the integral, which then reduces to $M = L\pi$.

The electromagnetic radiation reaching the earth is composed of a variety of wavelengths and so we must indicate whether we are describing *monochromatic* or *spectral* quantities. A subscript of λ, ν, or κ is often used to indicated where we are interested in wavelength, frequency or wavenumber. Radiance is simply the integral over all wavelengths (frequencies, wavenumbers) or monochromatic radiance, $L = \int_0^\infty L_\lambda d\lambda = \int_0^\infty L_\nu d\nu = \int_0^\infty L_\kappa d\kappa$.

The most fundamental radiation unit for satellite meteorology is monochromatic radiance, which is the energy per unit time per unit wavelength per unit solid angle crossing a unit area perpendicular to the beam. This is because many satellite instruments have a detector of a certain area whose output is proportional to the energy per unit time striking it.

Blackbody Radiation

A *blackbody* is a material where absorption is complete. The material is a perfect absorber in all wavelengths. Emission by a blackbody is the converse of absorption. Scientists used a cavity to create a blackbody that allowed them to determine a relationship between temperature and wavelength.

Envision a black body as a cavity with a very small entrance hole. Most of the radiant flux entering from outside will be trapped within the cavity. Repeated internal reflection will occur until the wall absorbs all of the fluxes. The probability of any entering flux escaping out of the cavity through the hole is so small that the cavity interior appears dark. Radiant flux emitted by a small area of the cavity is repeatedly reflected (absorbed and strengthened by new emissions). Eventually, the emission and absorption reach equilibrium with respect to the wall temperature.

When trying to derive a theoretical explanation for cavity radiation, Planck treated the atoms of the wall as tiny electromagnetic oscillators that each had a characteristic oscillation frequency. Planck assumed that the wall of the cavity could exchange energy with the radiation field inside the cavity in discrete bundles called quanta given by $\Delta E = h\nu$, where h is known as *Planck's constant* (6.6260755 x 10^{-34} J s). Using this assumption, Planck demonstrated that the radiance being emitted by a blackbody is given by $B_\lambda(T) = \dfrac{2hc^2\lambda^{-5}}{\exp(\frac{hc}{\lambda kT}) - 1}$, where k is *Boltsmann's constant* (1.380658 x 10^{-23} J K^{-1}).

Looking at Plank's curve, you can see that for any temperature T, there is one maximum. By differentiating the Planck function with respect to wavelength and setting the result equal to zero, we can obtain the wavelength of the maximum: $\lambda_m = a/T$, where a=2.897.9 x 10^{-3} m K. This relationship is called *Wein's displacement law*.

One of the important aspects of Planck's function is that it's value for a blackbody, when integrated over all wavelengths, is equal to σT^4. This result is called the *Stefan-Boltzmann law* and σ (5.67051 x 10^{-8} W m^{-2} K^{-4}) is the *Stefan-Boltzmann constant*.

There is an approximation to the Planck function that can be useful at millimeter and centimeter wavelengths, and for temperature like those encountered on the earth and in its atmosphere. This approximation is called *Rayleigh-Jeans approximation*, $B_\lambda(T) = \dfrac{c_1\lambda^{-5}}{\exp(\frac{c_2}{\lambda T}) - 1} \approx \dfrac{c_1}{c_2}\lambda^{-4}T$.

The two constants, c_1 and c_2, are called the first and second radiation constants (1.19104 x 10^{-16} W m^2 sr^{-1}, 1.4387 x 10^{-2} m K). This approximation says that in the microwave region of the EM spectrum, the radiance is proportional to the temperature. In remote sensing, it is often customary to divide the radiance by $(c_1/c_2)\lambda^{-4}$ to yield a *brightness temperature*.

Remember that a blackbody absorbs the maximum amount of radiation, and therefore must emit the maximum amount. The emissivity of a given wavelength, ε_λ, is the ratio of the emitting intensity to the Planck function, and is equal to the absorptivity, α_λ, for the medium under thermodynamic equilibrium, $\varepsilon_\lambda \equiv \alpha_\lambda$. This is called *Kirchoff's law*.

A medium with absorptivity α_λ absorbs only α_λ times the blackbody radiant intensity $B_\lambda(T)$, and therefore emits ε_λ times the blackbody radiant intensity. In a blackbody, the absorption and emission is maximum and so $\alpha_\lambda = \varepsilon_\lambda = 1$ for all wavelengths, which is why the material appears black. A gray body is characterized by incomplete absorption and emission, $\alpha_\lambda = \varepsilon_\lambda < 1$.

Emittance = $\varepsilon_\lambda \equiv \frac{emitted\_radiation(\lambda)}{B_\lambda(T)}$

Absorptance = $\alpha_\lambda \equiv \frac{absorbed\_radiation(\lambda)}{incident\_radiation(\lambda)}$

Reflectance = $\rho_\lambda \equiv \frac{reflected\_radiation(\lambda)}{incident\_radiation(\lambda)}$

Transmittance = $\tau_\lambda \equiv \frac{transmitted\_radiation(\lambda)}{incident\_radiation(\lambda)}$

2. Earth-Sun Relationship

The star of our solar system, the sun, is of about average mass, but below average in size. Nearly all of the energy that the earth receives and that drives the ocean and atmosphere comes from the sun. The distance between the earth and sun averages about 150 million kilometers (93 million miles). The great distance between the earth and sun, plus the earth's relatively small size compared to the sun allows us to assume that the sun's rays strike the earth in straight paths (irradiance vs. radiance).

The earth's axis of rotation is tilted 23 ½ degrees from the perpendicular to the earth's plane of revolution. The tilt is in the same direction throughout the year and is currently oriented so that the North Pole points to the North Star (Polaris). This means that the Northern Hemisphere is tilted toward the sun during the summer and away from the sun in winter months. This is the cause of our seasons.

The sun strikes the curved surface of the earth at different angles depending on the latitude of the observer.

The orbit of the earth takes approximately 365 days in an ellipse around the sun. The amount of solar radiant energy reaching the earth is determined primarily by the earth's rotation and it's orbit.

It is useful for remote sensing to know what the sun angle is at a given location for a given time. To this end we can define the solar position using the following terminology:

Solar declination – the latitude at which the sun is directly overhead at solar noon,
Zenith angle – the angle between a point directly overhead and the sun at solar noon,
Solar elevation (sun) angle – the angle of the sun above the horizon at solar noon.

To calculate the noon zenith angle (A), simply find the number of degrees of latitude separating the location receiving the direct vertical rays of the sun and location in question. The sun angle (B) is calculated by subtracting the zenith angle from 90 degrees.

You can approximate the solar declination using the following formula: $Solar\_declination = 23.5 * \sin(N)$ where N is the number of days to the closest equinox, expressed in degrees. By convention N is positive between March and September and negative from September to March.

Example:

What is the declination on April 20?

The closest equinox is on March 21, so N is 30. Using the above formula, the declination must be 11.75°.

The sun elevation angle is important because it affects the intensity of solar radiation received at the earth's surface. When the sun angle is large, the solar rays are more direct, are spread over a smaller surface area, and pass through less atmosphere, resulting in greater radiation per unit area.

If a beam of sunshine strikes the flat earth at a sun angle of 36 ½° above the horizon, the beam is actually spread out over 1.681 area units, which diminishes the maximum intensity of radiation to ~60%. For example, in the morning and evening, the amount of solar energy received per unit area is much less than at noon. This is why you are often warned to reduce sun exposure during mid-day hours.

The earth makes one rotation every 24 hours at a steady rate. We experience this rotation as day and night. At a given time only half of the earth is illuminated by the sun. The division between the light and dark halves of the earth is called the *circle of illumination*. This division runs through the poles during the spring and fall equinoxes.

The Solar Constant

The solar radiation reaching the earth originates (for our purposes) from a layer of the sun called the photosphere, which coincides with the visible disk of the sun. Although the sun is made up of gas, the photosphere is referred to as the surface of the sun. The photosphere is a relatively thin layer, about 500 km thick and has a radius of 6.96×10^5 km. The temperature of the photosphere varies from 8000 K in the lower layer to about 4000 K in the upper layer. Before reaching the earth the solar radiation must pass through the chromosphere and the corona.

The total amount of energy reaching the top of the atmosphere is called the *solar constant*. It is defined as the flux of solar energy (per unit time) across a surface of unit area normal to the solar beam at the mean distance between the sun and the earth.

The sun emits energy at the rate of 6.2×10^7 W m^{-2}. The total amount of energy reaching the top of the atmosphere varies slightly but can be considered to be 1370 W m^{-2}.

The Solar Spectrum

The *solar spectrum* is the distribution of the EM radiation emitted by the sun as a function of wavelength incident of the earth-atmosphere system. The solar irradiance incident at the top of the earth's atmosphere peaks in the visible portion of the spectrum near 0.46 μm (460 nm).

Solar Radiative Transfer through the Atmosphere

The *index of refraction*, *n*, of a substance is the ratio of the speed of light in a vacuum to the speed that EM radiation travels in that substance. At sea level, the index of refraction is approximately 1.0003. For most purposes, the speed of light in a vacuum can be used safely in the atmosphere. However, strong vertical gradients of atmospheric density and humidity can result in strong vertical gradient of *n*. This causes bending of EM rays and slight dislocation of satellite scan spots.

To describe radiative transfer through the atmosphere, let's consider the radiation that is incident on a differential volume (draw). The equation describes changes in radiance as the EM radiation passes through this volume. For the moment we are not going to consider polarization effects. So the changes that can occur to the radiation are: absorption, emission, scattering out of the beam, and scattering into the beam.