

1. A) Write down the equations of motion for both the east/west and north/south motion whereby the local acceleration is balanced by the Coriolis acceleration.

$$\begin{aligned} \text{east - west, } \frac{\partial u}{\partial t} &= fv \\ \text{north - south, } \frac{\partial v}{\partial t} &= -fu \end{aligned}$$

B) Write down the equations of motions for both the east/west and north/south motion whereby the pressure gradient is balanced by the Coriolis acceleration.

$$\begin{aligned} \text{east - west, } 0 &= -\frac{1}{\rho} \frac{\partial P}{\partial x} + fv \\ \text{north - south, } 0 &= -\frac{1}{\rho} \frac{\partial P}{\partial y} - fu \end{aligned}$$

C) Write an expression that describes the velocity as a function of time assuming that this pressure gradient is balanced only by the local acceleration and that the pressure gradient remains constant in time. Draw the result graphically.

Assume it is in east-west direction, and then the force balance would be:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x}$$

Also, we know that the pressure gradient remains constant in time, so we could get:

$$u = -\frac{1}{\rho} \frac{\partial P}{\partial x} * t + C_0, \text{ Where } C_0 \text{ is a constant}$$

2. A steady wind blows at 10 m/s to the north. The wind stress can be parameterized as $C_d * U^2$ where $C_d = .0015$

A) Write down the momentum equation assuming that the momentum balance is between the wind friction and the Coriolis force.

The momentum equation can be written vertically averaged over the Ekman layer

$$\frac{\tau^y}{\rho f} = u D_e$$

Where D_e is the Ekman Layer thickness

Or we can write the depth *dependent* equation:

$$fu = \frac{\partial \tau^y}{\partial z}$$

$$-fv = \frac{\partial \tau^x}{\partial z}$$

B) If this wind forcing occurred over a 100 km square box where $f = 10^{-4} \text{ s}^{-1}$ what would be the transport of water through each side of the box?

Flow is to the east. Flow vertically averaged over the Ekman layer on the north and south side of the box is zero. Flow through the east and west side of the box is:

$$uD_e \Delta x = \frac{\tau^y}{\rho f} \Delta x = \frac{.0015 * 10 * 10}{10^3 * 10^{-4}} 10^5 = 1.5 * 10^5 \text{ m}^3/\text{s}$$

C) Now consider the fact that the Coriolis parameter varies with latitude. How will the transport and Ekman depth vary with latitude? Is the flow divergent or non-divergent?

Both the Ekman transport and Ekman Depth will decrease with latitude. However, since the volume transport that enters the box on the west side is exactly equal to the volume transport that leaves on the east side—the flow is non-divergent.

A second way to think about this would be mathematically (of course!) the divergence of the horizontal flow is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

where u and v are the vertically averaged flow in the Ekman layer.

Since v is zero the second term is zero.

The first term is also zero because

$$\frac{\tau_y}{\rho f D_e} = u$$

and while f and D_e vary with y —they do not vary in x - so $\frac{\partial u}{\partial x} = 0$ and thus the flow is non-divergence

D) If the wind were from the west and we considered the fact that f changes with latitude would the Ekman flows be divergent or non-divergent? Would the flows lead to a change in the thickness of the mixed layer? Would it get thicker, thinner or stay the same?

Here the flow will be divergent, because the Ekman transport, which is now to the south, increases with decreasing latitude. Therefore the transport leaving the south side of the box is greater than the transport entering the north side of the box and the flow is divergent. The divergent surface flows produce an upwelling and this tends to decrease the thickness of the mixed layer.