1. A) Write down the equations of motion for both the east/west and north/south motion whereby the local acceleration is balanced by the Coriolis acceleration.

B) Write down the equations of motions for both the east/west and north/south motion whereby the pressure gradient is balanced by the Coriolis acceleration.

C) Write an expression that describes the velocity as a function of time assuming that this pressure gradient is balanced only by the local acceleration and that the pressure gradient remains constant in time. Draw the result graphically.

2. A steady wind blows at 10 m/s to the north. The wind stress can be parameterized as $C_d * U^2$ where $C_d = .0015$

A) Write down the momentum equation assuming that the momentum balance is between the wind friction and the Coriolis force.

B) If this wind forcing occurred over a 100 km square box where $f= 10^{-4}\ s^{-1}$ what would be the transport of water through each side of the box?

C) Now consider the fact that the Coriolis parameter varies with latitude. How will the transport and Ekman depth vary with latitude? Is the flow divergent or non-divergent?

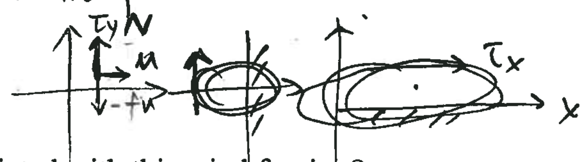
D) If the wind were from the west and we considered the fact that f changes with latitude would the Ekman flows be divergent or non-divergent? Would the flows lead to a change in the thickness of the mixed layer? Would it get thicker, thinner or stay the same?

3) A wind stress of 0.1 Pascals (or N/m^2) blowing to the north is applied to the surface ocean. The water is homogenous and the depth $H=ax$ where $a=10^{-3}$. Assume that the vertical eddy viscosity is constant and equal 10^{-2} m2/s.

A) Assuming that the flow in the off-shore region is an Ekman balance write the two terms that describe this momentum balance

The two terms are coriolis force and wind stress force. i.e.

$$0 = -fu + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$



B) What is the Ekman transport associated with this wind forcing?

According to ~~A~~ : $0 = -fu + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$

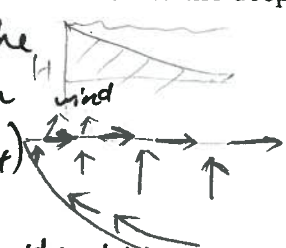
$$\Rightarrow fu = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \Rightarrow f\int u\,dz = \frac{1}{\rho}\int \frac{\partial \tau_y}{\partial z}dz = \frac{1}{\rho}\int \partial \tau_y$$

$$\Rightarrow f \cdot U_E = \frac{1}{\rho}[\tau_y^s - \tau_y^b] \Rightarrow U_E = \frac{1}{\rho f}\tau_y^s = \frac{0.1\ N\cdot m^{-2}}{10^3 \times 10^{-4}\ kg\cdot m^{-3}\cdot s^{-1}} = 1\ (m^2/s)$$

C) Describe how the flow varies between the shallow near-shore zone and the deeper offshore region.

Because near-shore, ~~there is bottom friction, so the~~ the depth is smaller, Ekman transport to the east is smaller than that of the deep offshore region. (i.e. $M_{near} < M_{off}$)

So because of the continuity equation ⇒ there will be upwelling flows, as showed on the right



D) Use a simple scaling to estimate the depth and cross-shore distance that the transition between the near-shore and off-shore flows might be (hint—it's not the Rossby radius).

The depths distinguishing near-shore and off-shore should be the Ekman layer depth i.e. D_e.

Excellent!

Also: $D_e = \pi\sqrt{\frac{2A_v}{f}} = 3.14 \cdot \sqrt{\frac{2\times 10^{-2}}{10^{-4}}} \simeq 44.4$ m.

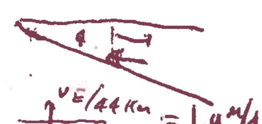
Also we know $H = ax \Rightarrow X_e = \frac{H}{a} = \frac{44.4m}{10^{-3}} = 4.44 \times 10^4$ m $= 44.4$ Km.

E) Based on your answer in A and C estimate the upwelling velocity.

From A, we know that : $M = \frac{1}{\rho f}\frac{\partial \tau_y}{\partial z}$

-3

So the transport: $U_E = M \cdot H = \frac{H}{\rho f}\cdot \tau_y$



$\frac{\tau_x}{\rho f}$

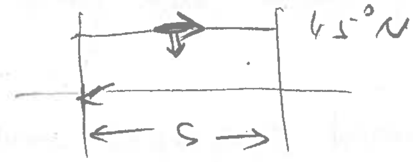
-2

So according the continuity equation:

$$\frac{\partial U_E}{\partial x} + (W_s - W_D) = 0 \Rightarrow W_D = \frac{\partial U_E}{\partial x} = \frac{\tau_y}{\rho f}\left(\frac{\partial H}{\partial x}\right)$$

$$\approx \frac{0.1}{10^3 \cdot 10^{-4}} \cdot 10^{-3} = 10^{-3}\ (m/s)$$

4) The mean wind speed in the North Atlantic at 45° N is to the east at 7 m/s and extends from the North American Continent to Europe over a distance of 5000 km.



A) Calculate the total Ekman transport associated with this wind forcing (units should be in Sv). when it gets into Ekman balance, i.e. $0 = fV + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$

$$\Rightarrow V_E = -\frac{1}{\rho f}(\tau_x^s - \tau_x^b) = -\frac{1}{\rho f}\tau_x = -\frac{\rho_a C_D w^2}{\rho f} \simeq -\frac{10^{-3}\cdot 49}{10^3 \times 1.0 \times 10^{-4}} = -0.47\ (m^2/s)$$

So the total transport : $V_{Tot,1} = V_E \cdot S = -0.47 \cdot 5000 \times 10^3 = -2.35 \times 10^6\ (m^3/s)$

$\simeq -2.35$ Sv

B) If the trade winds, at 20° have the same mean wind speed but blow in the opposite direction what is the divergence associated with this wind stress curl (Hint- just use the convergence of the two transports to estimate this)?

The same equation applied, $V_{Tot_2} \simeq$ ~~0.47~~ 5 Sv

so divergence would be $|V_{Tot_1}| + |V_{Tot_2}| = 7.35$ Sv

$= 7.35 \times 10^6\ m^3/s$

C) Calculate the rate that this divergence would thicken the upper layer.

The surface area of this area should be:

$$S = \frac{(45^{\circ} - 20^{\circ})}{90} \times \frac{\pi R}{2} \times 5000\,km = 1.39 \times 10^{13}\ (m^2)$$

So the rate : $w = \frac{|V_{Tot_1}| + |V_{tot_2}|}{S} = 5.28 \times 10^{-7}\ (m/s)$

D) What is the inertial period at these two latitudes?

At $45^{\circ}N$; The period $T = \frac{1}{f_1} = \frac{}{2 \times 7.27 \times 10^{-5} \times \sin 45} = 9.7 \times 10^3\ (s)$

At $20^{\circ}N$, The period $T = \frac{1}{f_2} = \frac{1}{2 \times 7.27 \times 10^{-5} \times \sin 20} = 2.0 \times 10^4\ (s)$

Problem 3.

a) If the wind stress along the New Jersey coast is as shown below, will there be upwelling or downwelling at the bottom of the Ekman layer (z = - h)? Why?

Upwelling, because the water is going to be deflected to the right (north hemisphere) away from the coast. This is going to cause a diverge near the coast and water has to come from below to replace the water going away from the shore.

b) What will be the average vertical velocity at z = - h if the wind stress is equal to 0.3 Nm-2? (Assume f = 0.0001/s)

$\int f u \, dz = + \frac{1}{\rho} \int \frac{\partial \tau}{\partial z} dz \Rightarrow f U_x = \frac{1}{\rho} \tau^y$ $\quad U_x \rightarrow$ Ekman transport.

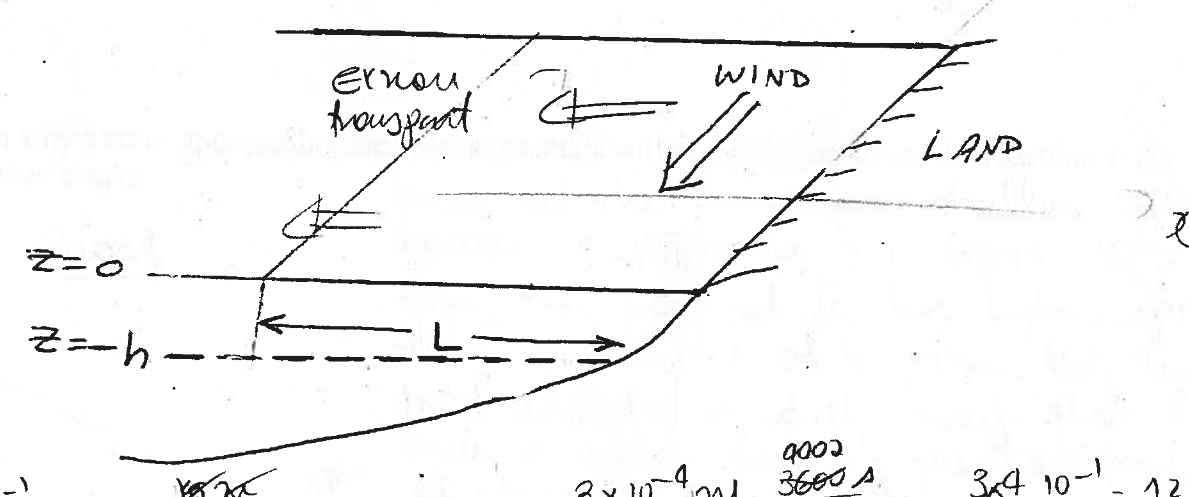
$|U_x| \rightarrow \frac{m^3}{s \cdot m}$. from the Continuity eq.: $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \rightarrow \int \frac{\partial u}{\partial x} dz = - \int \frac{\partial w}{\partial z} dz$

$\Rightarrow \frac{\partial}{\partial x} U_x = W_0 - W_{-h}$ and $W_0 = 0$ $\Rightarrow \frac{\Delta U_x}{\Delta x} = W_{-h} \Rightarrow$

$\Rightarrow W_{-h} = \frac{(\overset{0}{U_{shore}} - U_{x=L})}{L} \Rightarrow W_{-h} = \frac{U_{x=L}}{L} = \frac{\tau^y}{\rho f L} = \frac{0.3 \, N/m^2}{1000 \frac{kg}{m^3} \, 10^{-4} \frac{1}{s} \, 10^4 m} = 3 \times 10^{-4} \frac{m}{s} \approx 1.2 \frac{m}{h}$

c) How does this physical process impact biological processes?

The upwelling water usually comes from below the thermocline. This water is cold and nutrient-rich! Near the surface the phytoplankton takes up this nutrients and has enough light to reproduce.



$\frac{3 \times 10^{-1}}{10^3 \, 10^{-4} \, 10^4} \quad \frac{kg \, m}{s^2 m^2 \, \frac{kg}{m^3} \, \frac{1}{s} \, m} = 3 \times 10^{-4} \frac{m}{s} \frac{3600 s}{1 h} \quad 3 \times 4 \, 10^{-1} = 12 \times 10^{-1}$