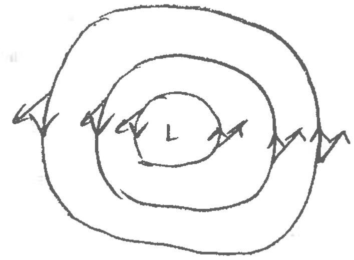
Answers for Recitation questions of Nov. 22nd & Nov. 29th. (By Peter Zhang)

(a)



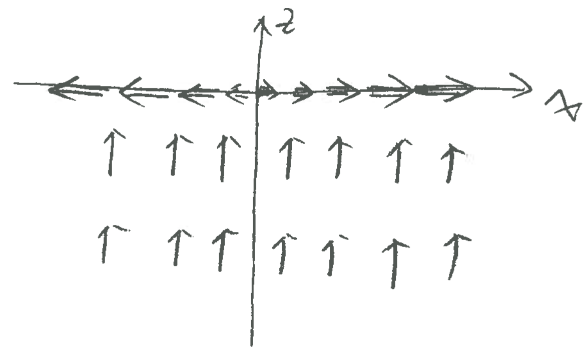
→ wind

→ Surface Ekman current

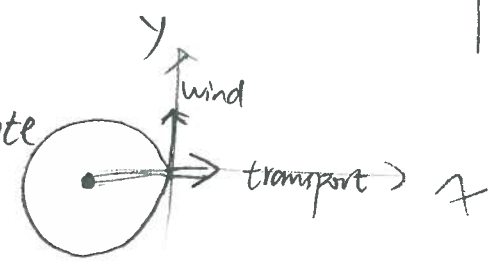
(c)

⇒ Ekman transport

↑ Associated upwelling speed.



(b) Here if we establish coordinate like shown on the right:

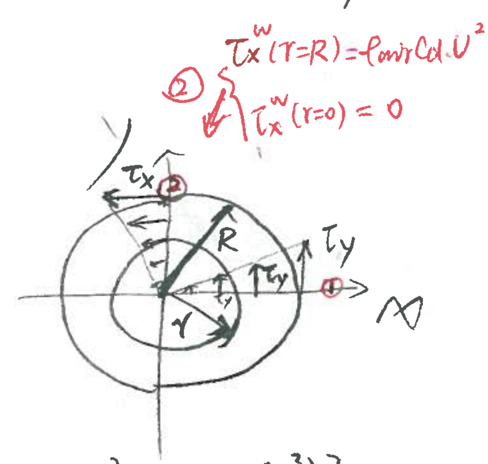


then: $T_x = \frac{1}{\rho f} \tau_y = \frac{1}{\rho f} (\rho_{air} \cdot C_d \cdot U^2) = \frac{1}{10^3 \cdot 10^{-4}} \cdot (1 \cdot 10^{-3} \cdot 50^2) = 25 \ m^2/s$

And the total transport along the whole circle would be:

$$\underbrace{T_x \cdot d}_{\text{circumference of this circle}} = T_x \cdot 2\pi R = 25 \times 2 \times 3.14 \times 30 \times 10^3 = 4.71 \times 10^6 \ m^3/s$$

(d) Since the surface shear stress varies linearly,



and ① $\boxed{\begin{cases} \tau_y^w (r=R) = \rho_{air} . C_d . U^2 & \leftarrow U = 50 \ m/s \text{ , outer edge} \\ \tau_y^w (r=0) = 0 & \leftarrow \text{No wind speed} \end{cases}}$

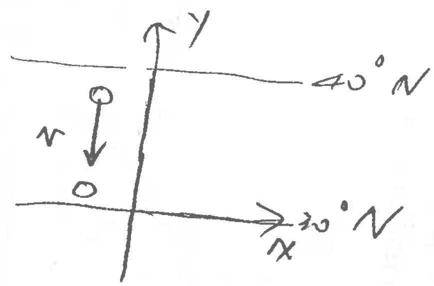
From the equation: $W_e (z=-D_E) = \frac{1}{\rho f} \left(\overset{①}{\frac{\partial \tau_y^w}{\partial x}} - \overset{②}{\frac{\partial \tau_x^w}{\partial y}} \right) = \frac{1}{\rho f} \left[\frac{\rho_{air} C_d \cdot U^2}{R} - \frac{(-\rho_{air} C_d U^2)}{R} \right]$

$$= \frac{1}{\rho f} \cdot \frac{2 \rho_{air} \cdot C_d \cdot U^2}{R} = \frac{1}{10^3 \cdot 10^{-4}} \cdot \frac{2 \times 1 \times 10^{-3} \times (50)^2}{30\ Km} = 1.67 \times 10^{-3}\ m/s.$$

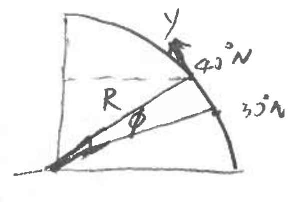
$$= 144\ m/day.$$

2) From the conservation of potential vorticity:

$$\frac{d}{dt}(P.V) = \frac{d}{dt}\left(\frac{\zeta + f}{H}\right) = 0.$$



(a).

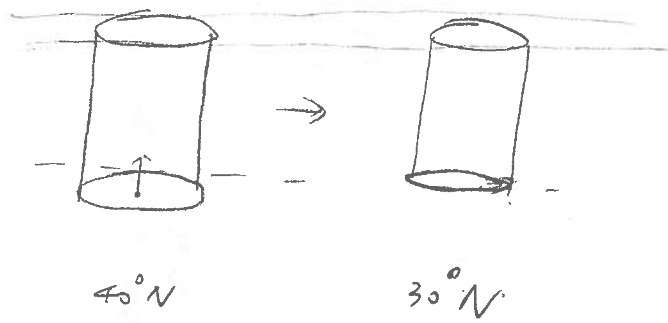


(b)

From the formula: $\left(\frac{b}{a}\right)' = \frac{b'a - a'b}{a^2}$

$$\Rightarrow \frac{\frac{d(\zeta+f)}{dt} \cdot H - \frac{dH}{dt} \cdot (\zeta + f)}{H^2} = 0.$$

$$\Rightarrow \boxed{\frac{dH}{dt}} = \frac{H}{\zeta + f} \cdot \frac{d(\zeta + f)}{dt}.$$



(c)

↓

$-W_e$(bottom)

$$\Rightarrow W_e(\text{bottom}) = \frac{-H}{\zeta + f} \cdot \left(\cancel{\frac{d\zeta}{dt}}^{=0} + \frac{df}{dt}\right) = -\frac{H}{\zeta + f} \cdot \frac{df}{dt}$$

plug in

$$\text{Also, } f = 2\Omega \sin\phi \Rightarrow \frac{df}{dt} = 2\Omega \cdot \cos\phi \cdot \frac{d\phi}{dt} = 2\Omega \cos\phi \cdot \frac{dy}{dt} \cdot \frac{1}{R} = \frac{2\Omega \cos\phi}{R} \cdot v.$$

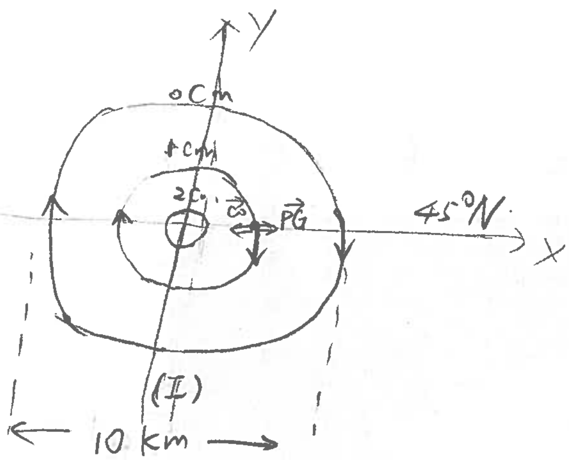
$$\Rightarrow W_e(\text{bottom}) = -\frac{H}{\zeta + f} \cdot \frac{2\Omega \cos\phi}{R} \cdot v.$$

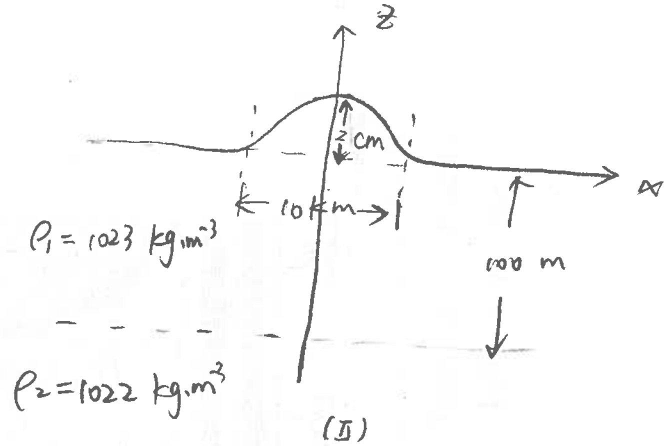
Here in this case: $\zeta = -0.1 f$; $H = 1000\ m$; $\Omega = 7.29 \times 10^{-5}\ rad/s$; $v = -0.5\ m/s$.
$\phi = 35°N$ $R = 6370\ Km$.

$$\Rightarrow W_e(\text{bottom}) = -\frac{H}{0.9 \sin\phi} \cdot \frac{\cos\phi}{R} \cdot v = +1.25 \times 10^{-4}\ m/s$$

rise upward.

3.

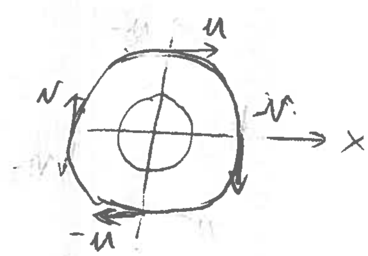




(a). Shown in figure (I)

$$f v = \frac{1}{\rho}\frac{\partial P}{\partial x} \Rightarrow v = \frac{1}{\rho f}\frac{\partial P}{\partial x} \overset{P=\rho g(h+\eta)}{=} \frac{g}{f}\cdot\frac{\partial \eta}{\partial x} = \frac{10}{10^{-4}}\cdot\frac{0-2\,cm}{5\ km} = -4\times10^{-3}\ (m/s)$$

(b). Relative vorticity $\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

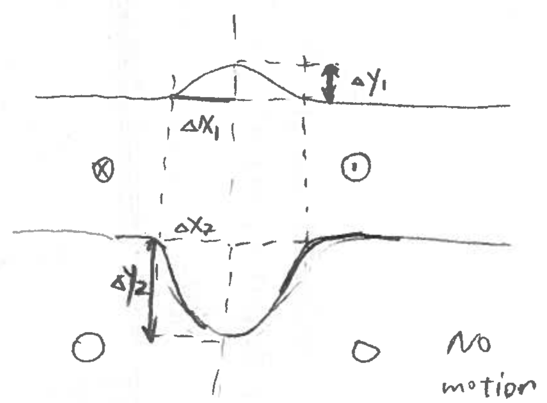


$$= \frac{-2v}{2R} - \frac{2u}{2R}$$

Also, $|u| = |v|$

$$\Rightarrow \zeta = -\frac{2v}{R} = -\frac{2\times 4\times10^{-3}}{5\ km} = -1.6\times10^{-6}\ (1/s)$$

(c). Using Margule's equation:



$$v_2 = -v_1\cdot\left(\frac{\rho_1}{\rho_2-\rho_1}\right)$$

$$\Rightarrow \frac{\Delta y_2}{\Delta x_2} = -\frac{\Delta y_1}{\Delta x_1}\cdot\frac{1022\ kg\cdot m^{-3}}{(1023-1022)\ kg\cdot m^{-3}}$$

$$\Rightarrow \Delta y_2 = -\frac{\Delta x_2}{\Delta x_1}\cdot\Delta y_1\cdot 1022$$

Also, $\Delta x_1 = \Delta x_2 \Rightarrow \Delta y_2 = -1022\,\Delta y_1 = -1022\times 2\ cm = -2044\ cm$

$= -20.44\ m.$

(d) $\uparrow f$ $45°N \rightarrow 30°N$

$f \downarrow$

$\frac{d}{dt}\left(\frac{\zeta+f}{H}\right)=0 \Rightarrow$ Either $\zeta \uparrow$, if H constant

Or $H \downarrow$, if ζ constant.

4) (a) $\beta M_y = \frac{\partial \tau_y^w}{\partial x} - \frac{\partial \tau_x^w}{\partial y} = \mathrm{Curl}(\vec{\tau}^w)$

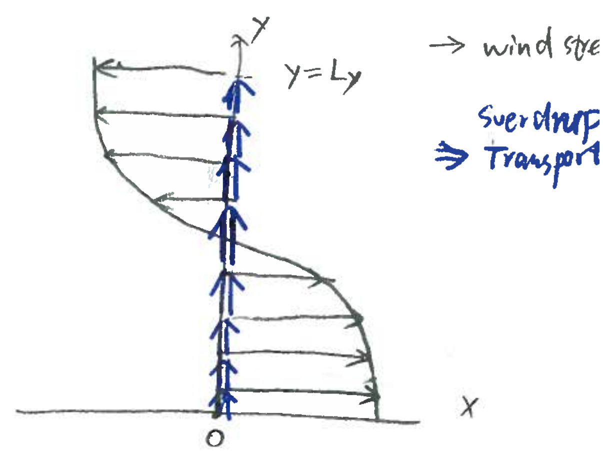
$$\Rightarrow M_y = \frac{1}{\beta}\left(-\frac{\partial \tau_x^w}{\partial y}\right) = \frac{1}{\beta}\cdot\left[+\,0.2\cdot\frac{\pi}{L_y}\sin\left(\frac{\pi\cdot y}{L_y}\right)\right] = \frac{0.2\cdot\pi}{\beta\cdot L_y}\sin\left(\frac{\pi}{L_y}\cdot y\right)$$

Also, $M_y = \rho_0\cdot\int_{-H}^{0} v\,dz = \rho_0\cdot\bar{v}\cdot H$

$\uparrow$

Assume the vertically averaged velociey. $\bar{v}$.

$$\Rightarrow \bar{v} = \frac{M_y}{\rho_0 H} = \frac{0.2\pi}{\rho_0 H\cdot\beta\cdot L_y}\cdot\sin\left(\frac{\pi}{L_y}\cdot y\right)$$



when y varies from $y=0$ to $y=L_y$

$\bar{v}$ is positive all the time

(b) Ignoring friction, we would have:

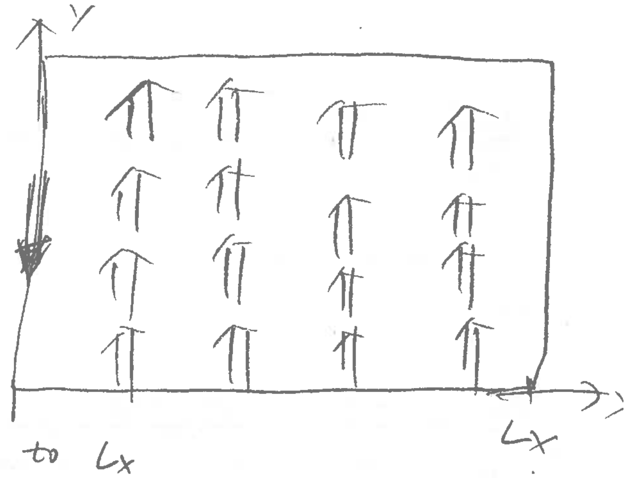
$\beta M_y = \mathrm{curl}(\vec{\tau}^w)$

Also, $\Delta Q = \Delta\left[\frac{\zeta+f}{H}\right] = \frac{\Delta\zeta}{H} \propto \Delta\,\mathrm{curl}(\vec{\tau}^w)$

$\Rightarrow \Delta(\beta M_y) = \Delta(\mathrm{curl}\,\vec{\tau}^w) \propto \frac{\Delta\zeta}{H}$

$\Rightarrow \Delta\bar{V} \propto \Delta\zeta \propto \Delta Q$

(C) Suppose the return flow is along the western boundary (~~only~~ assumption)



then the transport would be the integral of M_y ~~ove~~ along the x direction from 0 to L_x

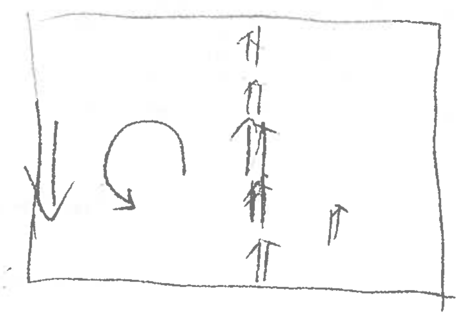
(Conservation ⇒ ~~All the flow toward North~~ has to equal all the flow toward north)

$$\Rightarrow \quad M_{total(y)} = \int_0^{L_x} M_y \cdot dx = \int_0^{L_x} \frac{1}{\beta} \cdot \mathrm{curl}(\vec{\tau}^w)\, dx = \frac{L_x}{\beta} \cdot \mathrm{curl}(\vec{\tau}^w)$$

$$= \frac{L_x}{\beta} \cdot \left(-\frac{\partial \tau_x^w}{\partial y}\right) = \frac{0.2\pi \cdot L_x}{\beta L_y} \cdot \sin\left(\frac{\pi}{L_y} \cdot y\right)$$

(d). Western Boundary Wall.

why ?

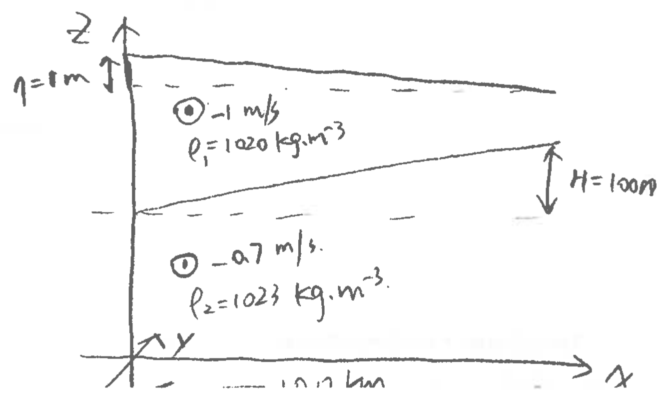


Balance of vorticity input to the whole system.

So friction ↻ ← Sink of vorticity

Could balance wind curl ↺ ← Source of vorticity

5) a) Establish one coordinate like shown on right:



For surface geostrophic current:

$$f v_1 = \frac{1}{\rho_1}\frac{\partial P}{\partial x} \Rightarrow v = \frac{g}{f}\frac{\partial \eta}{\partial x} = \frac{10}{10^{-4}} \cdot \frac{-1\,m}{100\,Km} = -1\ (m/s)$$

For lower layer geostrophic current.

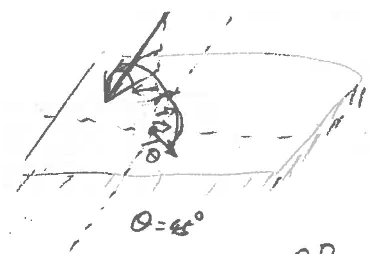
$$f v_2 = \frac{1}{\rho}\frac{\partial P}{\partial x} \Rightarrow \rho_2 f v_2 = \frac{\partial P_1}{\partial x} + \frac{\rho_2 - \rho_1}{\Delta x} g \Delta z = \rho_1 g \, i_1 + \Delta\rho\, g \, i_2$$

$$\Rightarrow \quad v_2 = \frac{\rho_1 g i_1 + \Delta\rho g i_2}{\rho_2 f} = \frac{1020 \times 10 \times \left(-\frac{1m}{100km}\right) + 3 \cdot 10 \cdot \frac{100m}{100km}}{1023 \cdot \times 10^{-4}}$$

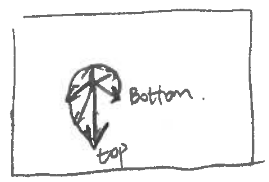
$$= -0.7 \ m/s.$$

(b). Bottom Ekman Layer.

Shown on the right



OR.



c). If the pressure gradient is exactly canceled. in the lower layer, which means no motion is lower layer, then:

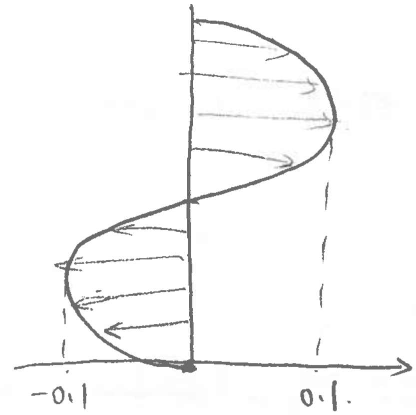
From Margule's equation:

$$i_2 = -i_1 \cdot \left(\frac{\rho_1}{\rho_2 - \rho_1}\right) = -\frac{1}{100km} \cdot \frac{1020}{3} = \frac{340 \ m}{100 \ km}$$

$$\Rightarrow H_1 = i_2 \cdot \Delta X = \frac{340m}{100km} * 100 \ km = 340 \ m.$$

d). There would be no flow in the lower layer, so the Bottom Ekman layer would not exist. (or the flow is zero).

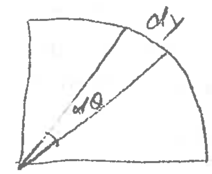
6). $\tau_x = 0.1 \cdot \sin(\text{latitude}/10)$



a). From : $W_e(z=-D_E) = \frac{1}{\rho f}\left(\frac{\partial \tau_y^w}{\partial x} - \frac{\partial \tau_x^w}{\partial y}\right)$

$= \frac{1}{\rho f} \cdot \left(-\frac{\partial \tau_x}{\partial y}\right)$

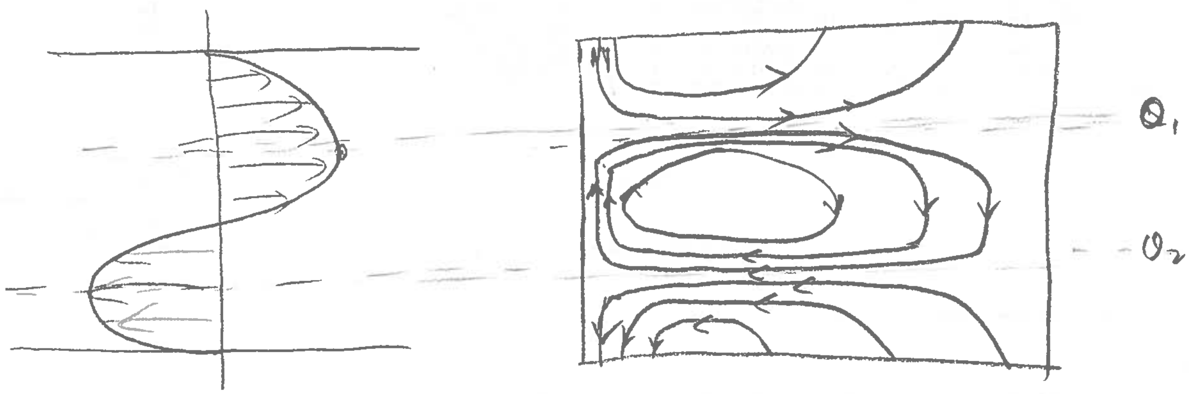
$= \frac{1}{\rho f} \cdot \left[-\frac{0.1}{10} \cdot \cos(\text{latitude}/10) \cdot \frac{d(\text{latitude})}{dy}\right. \rightarrow \frac{1}{R}$



$= -\frac{10^{-2}}{\rho R f} \cos(\text{latitude}/10)$ ✓

$\frac{dy}{d\theta} = \frac{dx}{d(\text{latitude})} = R$.

b).



$\theta_1 = \frac{3}{2}\pi \cdot 10 = 15\pi \approx 47.1^\circ$

$\theta_2 = \frac{\pi}{2} \cdot 10 = 5\pi \approx 15.7^\circ$

c). $\beta M_y = \text{curl}(\vec{\tau}^w) \Rightarrow M_y = \frac{1}{\beta} \text{curl}\, \vec{\tau}^w = -\frac{1}{\beta} \frac{\partial \tau_x^w}{\partial y} = -\frac{10^{-2}}{R} \cos(\text{latitude}/10)$

when $\text{Curl}(\vec{\tau}^w)$ is maximum $\Rightarrow$ M_y maximum.

when - - - is minimum $\Rightarrow$ M_y minimum.

Proportional