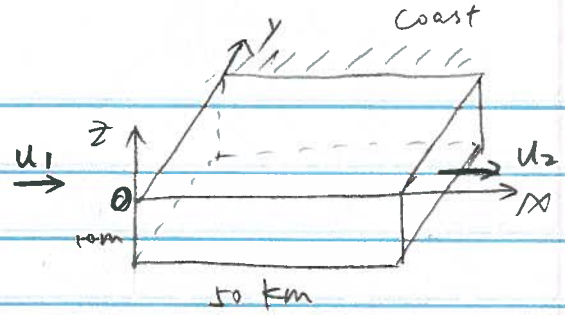
HW 2 Solutions:

1.

The figure is shown on the right.



a. Givens: ① $L = 10\ km$; $u(x) = 0.10 + 0.05 * \exp(-x/L)$ (m/s)

② $v = 0$ (No y-directional velocity assumption)

③ $w(z=0) = w_0 = 0$ (No vertical velocity on the surface)

④ ρ = constant.

From the continuity equation (ρ=constant):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Since $v=0$ $\Rightarrow$ $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$.

Calculate vertical integral to both sides from bottom ($z=-h$) to surface ($z=0$)

$$\int_{-h}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) dz = 0$$

$$\Rightarrow \int_{-h}^{0} \frac{\partial u}{\partial x} dz + \underbrace{\int_{-h}^{0} \frac{\partial w}{\partial z} dz}_{\int_{w_{-h}}^{w_0} dw \, = \, w_0 - w_{-h}} = 0$$

Also, $\frac{\partial u}{\partial x} = 0.05 \cdot (-\frac{1}{L}) \cdot \exp(-x/L)$ (not varying with z)

Put it into above equation, then:

$$\frac{\partial u}{\partial x} \cdot \int_{-h}^{0} dz + (w_0 - w_{-h}) = 0$$

Also, $w_0 = 0$ $\Rightarrow$ $\boxed{w_{-h} = \frac{\partial u}{\partial x} \cdot [0-(-h)] = h \cdot \frac{\partial u}{\partial x} = 0.05 \cdot (-\frac{h}{L}) \exp(-x/L)}$

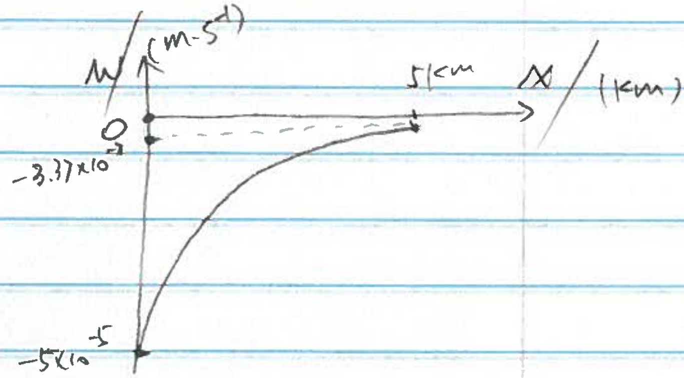
b. Put the values: $h = 10\ m$, $L = 10\ km$ into the above equation, then:

$$w_{-h} = -0.05 \cdot \frac{10\ m}{10\ km} \cdot \exp(-x/10km) = -5.0 \times 10^{-5} \cdot \exp(-\frac{x}{10^4})$$

when $x=0$ (left boundary), $W_n(x=0) = -5\times10^{-5}$ m/s

when $x = 50$ km (right boundary), $W_n(x=50km) \approx -3.37\times10^{-7}$ m/s.

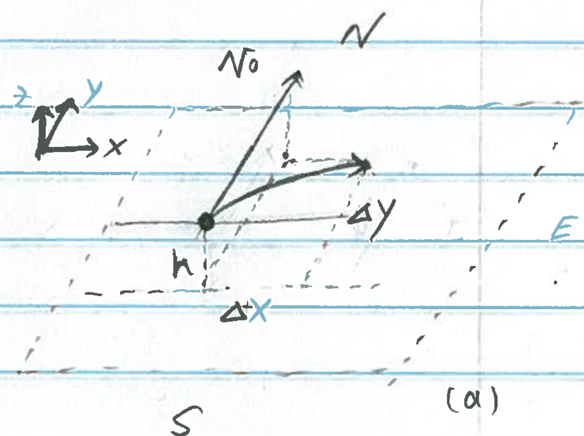
And the magnitude of W_n decrease expotentially with x, like shown on the figure below.



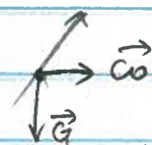
2. ① Gravity & coriolis force

②

The forces and travelling trajectory coordinates are Showing on the right.



(a)



(b).

Based on the given information and the coodinate, the following equations could be given:

$$\begin{cases} \dfrac{du}{dt} = f\cdot v_0 \\ \dfrac{dv}{dt} = f u_0 \\ \dfrac{dw}{dt} = -g. \end{cases} \quad \text{or} \quad \begin{cases} \Delta x = \frac{1}{2}(f v_0)t^2 + u_0 t \\ \Delta y = \frac{1}{2}(f u_0)t^2 + v_0 t \\ \Delta z = h = \frac{1}{2}gt^2 \end{cases}$$

where u_0, v_0 are the initial velocity of the luggage, f is coriolis parameter, g is gravitational acceleration.

③. Solving the above equations:

From givens: $v_0 = 600 \text{ km/hr} = 600 \cdot \frac{1000 \text{ m}}{3600 \text{ s}} = 166.7 \text{ m/s}$

$u_0 = 0,$

$h = 4\times10^4 \text{ ft} = 4\times10^4 \times 0.3048 \text{ m} = 12192 \text{ m}.$

$f = 2\Omega \cdot \sin\phi = 2\times7.27\times10^{-5} \times \sin 43^\circ = 9.92\times10^{-5} \text{ s}^{-1}$

$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \cong 49.88 \text{ s}.$

$\Rightarrow \Delta x = \frac{1}{2}\cdot(f v_0)\cdot t^2 = \frac{1}{2}\times9.92\times10^{-5}\times166.7\times49.88^2 = 20.57 \text{ (m)}$

$\Delta y = 0 + v_0 t = 166.7\times49.88 \cong 8313 \text{ (m)}$

So according to the established coordinate, we could find out that: the luggage would be 20.57 m on the East. and 8313 m on the North. of the dropping point