Question 3. Solution:

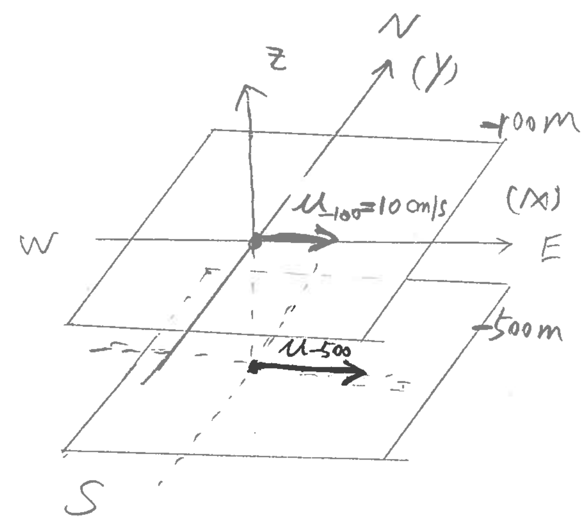
Given information:

$f = 10^{-4}\ s^{-1}$

$u(z=-100) = u_{-100} = 10\ cm/s = 0.1\ m/s$.

$\frac{dT}{dy} = \frac{-10\ K}{40\ km} = -2.5 \times 10^{-4}\ K/m$

$\rho = 1000 - 0.25\,T$ (linearized equation of state)



From the hint provided, we use the thermal wind equation:

$$\begin{cases} \frac{\partial u}{\partial z} = \frac{g}{f\rho_0} \cdot \frac{\partial \rho}{\partial y} & \text{--- ①} \\ \frac{\partial v}{\partial z} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x} & \text{--- ②} \end{cases}$$

So now we are required to calculate $u(z=-500)$, $u(z=-100)$ is already known, so only if we can calculate $\frac{\partial u}{\partial z}$, then it could get solved

So calculate vertical integral $\left(\int_{-500}^{-100} \cdots dz\right)$ to the ①, then we can get:

$$\int_{-500}^{-100} \frac{\partial u}{\partial z}\,dz = \int_{-500}^{-100} \frac{g}{f\rho_0} \cdot \frac{\partial \rho}{\partial y} \cdot dz \quad \text{--- ③}$$

And. $\frac{\partial \rho}{\partial y} = \frac{\partial}{\partial y}(1000 - 0.25T) = -0.25\frac{\partial T}{\partial y} = -0.25 \times (-2.5 \times 10^{-4}) = 6.25 \times 10^{-5}$ (kg/m^4)

Also, $g = 10\ m/s^2$; $f = 10^{-4}\ s^{-1}$; ρ_0 is the density average, and we can use $\rho_0 = 1000\ kg/m^3$ approximately. (Also, since ρ_0 appears in the denominator, so that would not make too much difference)

Since they are all constants here, so we put them outside of the integral.

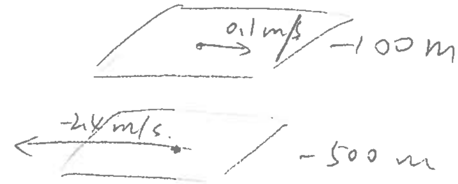
So, from ③ $\Rightarrow \int_{u(z=-500)}^{u(z=-100)} du = \frac{g}{f\rho_0}\frac{\partial \rho}{\partial y} \int_{-500}^{-100} dz$

$$\Rightarrow \quad u_{-100} - u_{-500} = \frac{g}{f\rho_0}\frac{\partial \rho}{\partial y}\cdot\left[-100-(-500)\right]$$

$$= \frac{10}{10^{-4}\times 1000} \times 6.25\times 10^{-5} \times 400.$$

$$\Rightarrow \quad u_{-500} = u_{-100} - 2.5 = 0.1 - 2.5 = -2.4 \ (m/s)$$

Since u_{-500} is negative, so the $z=-500$ layer fluid would flow to the west, with the magnitude of 2.4 m/s.



Since $\frac{\partial \rho}{\partial x} = \frac{\partial}{\partial x}(1000-0.25T) = -0.25\frac{\partial T}{\partial x} = 0$, then $\frac{\partial v}{\partial z} = 0$.

Also, $v_{-100} = 0 \Rightarrow v_{-500} = 0$

If the temperature decrease ---- to the $\boxed{\text{WEST}}$, as initially indicated, then: $\frac{dT}{dx} = \frac{10\,K}{40\,km} = 2.5\times 10^{-4}$ K/m

Use ②, calculate vertical integral:

$$\int_{-500}^{-100} \frac{\partial v}{\partial z}\cdot dz = \int_{-500}^{-100} -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}\, dz \quad \text{--- ④}$$

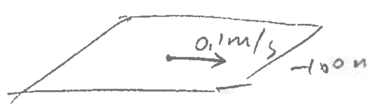
$$\frac{\partial \rho}{\partial x} = \frac{\partial}{\partial x}(1000-0.25T) = -0.25\cdot\frac{\partial T}{\partial x} = -6.25\times 10^{-5} \ (kg/m^4)$$

From ④ $\Rightarrow \quad v_{-100} - v_{-500} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}\int_{-500}^{-100} dz$

$$\Rightarrow \quad v_{-500} = v_{-100} + \frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}\left[(-100)-(-500)\right] = 0 + \frac{10}{10^{-4}\times 1000}\times(-6.25\times 10^{-5})\times 400$$

$$= -2.5 \ (m/s)$$

From ③ $\Rightarrow \quad u_{-100} - u_{-500} = \int_{-500}^{-100} \frac{g}{f\rho_0}\frac{\partial \rho}{\partial y}\, dz$



Since $\frac{\partial \rho}{\partial y} = 0 \Rightarrow u_{-500} = u_{-100} = 0.1$ m/s.

So: $\begin{cases} u_{-500} = 0.1 \text{ m/s} \\ v_{-500} = -2.5 \text{ m/s} \end{cases}$, like shown on the right.